CHAPTER 1 FUNCTIONS

1.1 FUNCTIONS AND THEIR GRAPHS

1. domain $= (-\infty, \infty)$; range $= [1, \infty)$

2. domain $= [0, \infty)$; range $= (-\infty, 1]$

3. domain $= [-2, \infty)$; y in range and $y = \sqrt{5x+10} \geq 0 \Rightarrow$ y can be any positive real number $\Rightarrow$ range $= [0, \infty)$.

4. domain $= (-\infty, 0] \cup [3, \infty)$; y in range and $y = \sqrt{x^2 - 3x} \geq 0 \Rightarrow$ y can be any positive real number $\Rightarrow$ range $= [0, \infty)$.

5. domain $= (-\infty, 3) \cup (3, \infty)$; y in range and $y = \frac{4}{3-t}$, now if $t < 3 \Rightarrow 3 - t > 0 \Rightarrow \frac{4}{3-t} > 0$, or if $t > 3 \Rightarrow 3 - t < 0 \Rightarrow \frac{4}{3-t} < 0 \Rightarrow$ y can be any nonzero real number $\Rightarrow$ range $= (-\infty, 0) \cup (0, \infty)$.

6. domain $= (-\infty, -4) \cup (-4, 4) \cup (4, \infty)$; y in range and $y = \frac{2}{t^2-16}$, now if $t < -4 \Rightarrow t^2 - 16 > 0 \Rightarrow \frac{2}{t^2-16} > 0$, or if $-4 < t < 4 \Rightarrow -16 \leq t^2 - 16 < 0 \Rightarrow -\frac{2}{16} \geq \frac{2}{t^2-16}$, or if $t > 4 \Rightarrow t^2 - 16 > 0 \Rightarrow \frac{2}{t^2-16} > 0 \Rightarrow$ y can be any nonzero real number $\Rightarrow$ range $= (-\infty, -\frac{1}{8}] \cup (0, \infty)$.

7. (a) Not the graph of a function of x since it fails the vertical line test.
   (b) Is the graph of a function of x since any vertical line intersects the graph at most once.

8. (a) Not the graph of a function of x since it fails the vertical line test.
   (b) Not the graph of a function of x since it fails the vertical line test.

9. base $= x$; $(\text{height})^2 + \left(\frac{x}{2}\right)^2 = x^2 \Rightarrow$ height $= \frac{\sqrt{3}}{2}x$; area is $a(x) = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(x)\left(\frac{\sqrt{3}}{2}x\right) = \frac{\sqrt{3}}{4}x^2$; perimeter is $p(x) = x + x + x = 3x$.

10. s = side length $\Rightarrow s^2 + s^2 = d^2 \Rightarrow s = \frac{d}{\sqrt{2}}$; and area is $a = s^2 \Rightarrow a = \frac{1}{2}d^2$

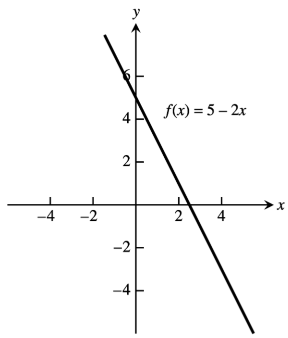
11. Let D = diagonal length of a face of the cube and ℓ = the length of an edge. Then $\ell^2 + D^2 = d^2$ and $D^2 = 2\ell^2 \Rightarrow 3\ell^2 = d^2 \Rightarrow \ell = \frac{d}{\sqrt{3}}$. The surface area is $6\ell^2 = \frac{6d^2}{3} = 2d^2$ and the volume is $\ell^3 = \left(\frac{d^2}{3}\right)^{3/2} = \frac{d^3}{3\sqrt{3}}$.

12. The coordinates of P are $\left(x, \sqrt{x}\right)$ so the slope of the line joining P to the origin is $m = \frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}}\ (x > 0)$. Thus, $\left(x, \sqrt{x}\right) = \left(\frac{1}{m^2}, \frac{1}{m}\right)$.

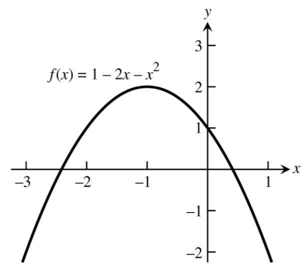
13. $2x + 4y = 5 \Rightarrow y = -\frac{1}{2}x + \frac{5}{4}$; $L = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + (-\frac{1}{2}x + \frac{5}{4})^2} = \sqrt{x^2 + \frac{1}{4}x^2 - \frac{5}{4}x + \frac{25}{16}}$
$= \sqrt{\frac{5}{4}x^2 - \frac{5}{4}x + \frac{25}{16}} = \sqrt{\frac{20x^2 - 20x + 25}{16}} = \frac{\sqrt{20x^2 - 20x + 25}}{4}$

14. $y = \sqrt{x-3} \Rightarrow y^2 + 3 = x$; $L = \sqrt{(x-4)^2 + (y-0)^2} = \sqrt{(y^2 + 3 - 4)^2 + y^2} = \sqrt{(y^2 - 1)^2 + y^2}$
$= \sqrt{y^4 - 2y^2 + 1 + y^2} = \sqrt{y^4 - y^2 + 1}$

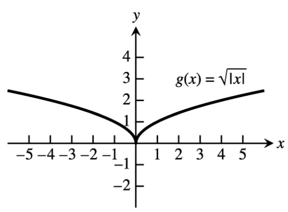
15. The domain is $(-\infty, \infty)$.



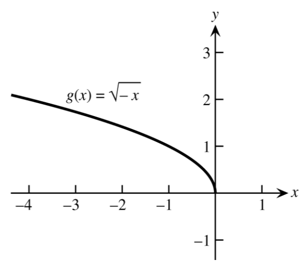
16. The domain is $(-\infty, \infty)$.



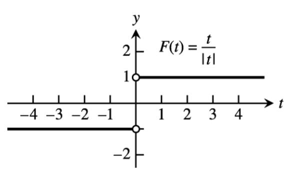
17. The domain is $(-\infty, \infty)$.



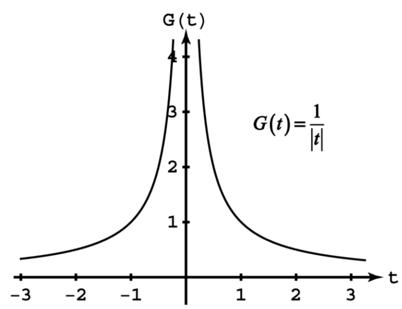
18. The domain is $(-\infty, 0]$.



19. The domain is $(-\infty, 0) \cup (0, \infty)$.



20. The domain is $(-\infty, 0) \cup (0, \infty)$.

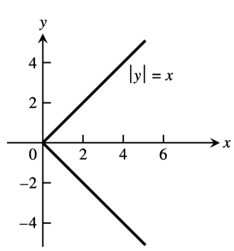


21. The domain is $(-\infty, -5) \cup (-5, -3] \cup [3, 5) \cup (5, \infty)$

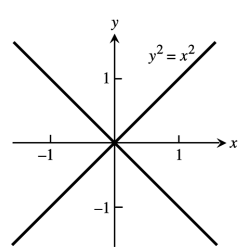
22. The range is $[2, 3)$.

23. Neither graph passes the vertical line test

(a)

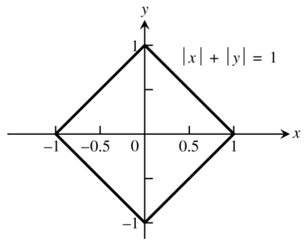


(b)

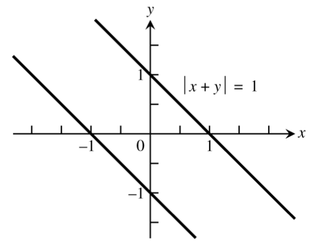


24. Neither graph passes the vertical line test

(a)



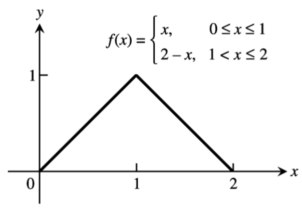
(b)



$$|x+y|=1 \Leftrightarrow \left\{\begin{matrix} x+y=1 \\ \text{or} \\ x+y=-1 \end{matrix}\right\} \Leftrightarrow \left\{\begin{matrix} y=1-x \\ \text{or} \\ y=-1-x \end{matrix}\right\}$$

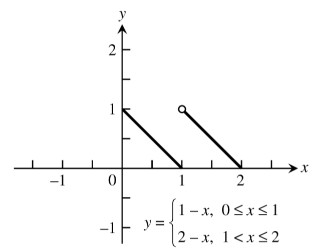
25.

x	0	1	2
y	0	1	0

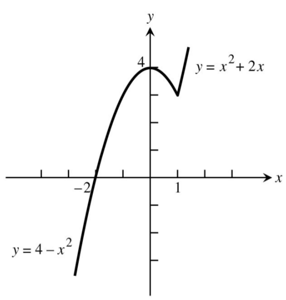


26.

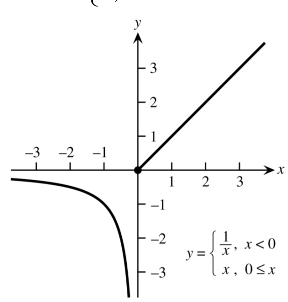
x	0	1	2
y	1	0	0



27. $F(x)=\begin{cases} 4-x^2, & x \le 1 \\ x^2+2x, & x>1 \end{cases}$



28. $G(x)=\begin{cases} \frac{1}{x}, & x<0 \\ x, & 0 \le x \end{cases}$



29. (a) Line through (0, 0) and (1, 1): $y=x$; Line through (1, 1) and (2, 0): $y=-x+2$

$$f(x)=\begin{cases} x, & 0 \le x \le 1 \\ -x+2, & 1<x \le 2 \end{cases}$$

(b) $$f(x)=\begin{cases} 2, & 0 \le x < 1 \\ 0, & 1 \le x < 2 \\ 2, & 2 \le x < 3 \\ 0, & 3 \le x \le 4 \end{cases}$$

30. (a) Line through (0, 2) and (2, 0): $y=-x+2$

Line through (2, 1) and (5, 0): $m=\frac{0-1}{5-2}=\frac{-1}{3}=-\frac{1}{3}$, so $y=-\frac{1}{3}(x-2)+1=-\frac{1}{3}x+\frac{5}{3}$

$$f(x)=\begin{cases} -x+2, & 0<x \le 2 \\ -\frac{1}{3}x+\frac{5}{3}, & 2<x \le 5 \end{cases}$$

(b) Line through (–1, 0) and (0, –3): $m = \frac{-3-0}{0-(-1)} = -3$, so $y = -3x - 3$

Line through (0, 3) and (2, –1): $m = \frac{-1-3}{2-0} = \frac{-4}{2} = -2$, so $y = -2x + 3$

$$f(x) = \begin{cases} -3x-3, & -1 < x \le 0 \\ -2x+3, & 0 < x \le 2 \end{cases}$$

31. (a) Line through (–1, 1) and (0, 0): $y = -x$

Line through (0, 1) and (1, 1): $y = 1$

Line through (1, 1) and (3, 0): $m = \frac{0-1}{3-1} = \frac{-1}{2} = -\frac{1}{2}$, so $y = -\frac{1}{2}(x-1) + 1 = -\frac{1}{2}x + \frac{3}{2}$

$$f(x) = \begin{cases} -x & -1 \le x < 0 \\ 1 & 0 < x \le 1 \\ -\frac{1}{2}x + \frac{3}{2} & 1 < x < 3 \end{cases}$$

(b) Line through (–2, –1) and (0, 0): $y = \frac{1}{2}x$

Line through (0, 2) and (1, 0): $y = -2x + 2$

Line through (1, –1) and (3, –1): $y = -1$

$$f(x) = \begin{cases} \frac{1}{2}x & -2 \le x \le 0 \\ -2x+2 & 0 < x \le 1 \\ -1 & 1 < x \le 3 \end{cases}$$

32. (a) Line through $\left(\frac{T}{2}, 0\right)$ and $(T, 1)$: $m = \frac{1-0}{T-(T/2)} = \frac{2}{T}$, so $y = \frac{2}{T}\left(x - \frac{T}{2}\right) + 0 = \frac{2}{T}x - 1$

$$f(x) = \begin{cases} 0, & 0 \le x \le \frac{T}{2} \\ \frac{2}{T}x - 1, & \frac{T}{2} < x \le T \end{cases}$$

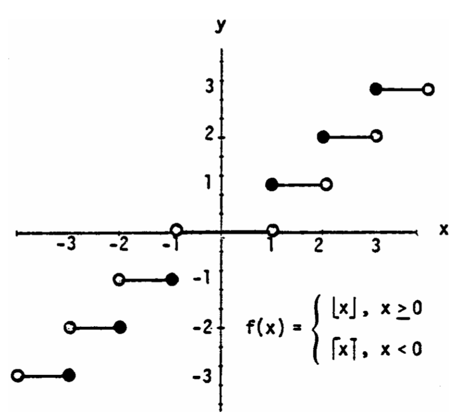
(b) $$f(x) = \begin{cases} A, & 0 \le x < \frac{T}{2} \\ -A, & \frac{T}{2} \le x < T \\ A, & T \le x < \frac{3T}{2} \\ -A, & \frac{3T}{2} \le x \le 2T \end{cases}$$

33. (a) $\lfloor x \rfloor = 0$ for $x \in [0, 1)$ (b) $\lceil x \rceil = 0$ for $x \in (-1, 0]$

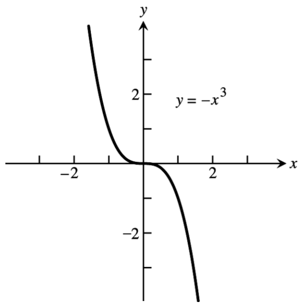
34. $\lfloor x \rfloor = \lceil x \rceil$ only when x is an integer.

35. For any real number x, $n \le x \le n+1$, where n is an integer. Now: $n \le x \le n+1 \Rightarrow -(n+1) \le -x \le -n$. By definition: $\lceil -x \rceil = -n$ and $\lfloor x \rfloor = n \Rightarrow -\lfloor x \rfloor = -n$. So $\lceil -x \rceil = -\lfloor x \rfloor$ for all real x.

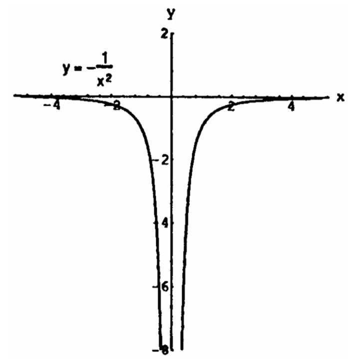
36. To find $f(x)$ you delete the decimal or fractional portion of x, leaving only the integer part.



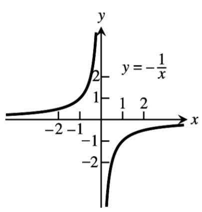
37. Symmetric about the origin
Dec: $-\infty < x < \infty$
Inc: nowhere



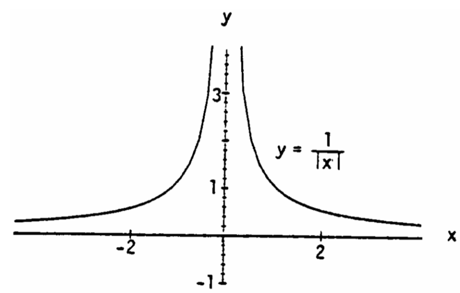
38. Symmetric about the y-axis
Dec: $-\infty < x < 0$
Inc: $0 < x < \infty$



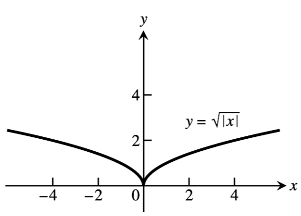
39. Symmetric about the origin
Dec: nowhere
Inc: $-\infty < x < 0$
$0 < x < \infty$



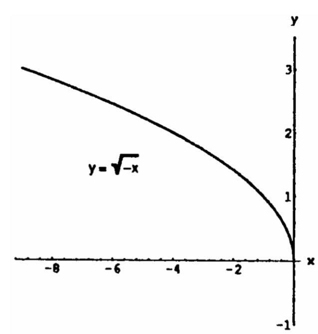
40. Symmetric about the y-axis
Dec: $0 < x < \infty$
Inc: $-\infty < x < 0$



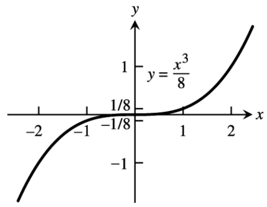
41. Symmetric about the y-axis
Dec: $-\infty < x \le 0$
Inc: $0 \le x < \infty$



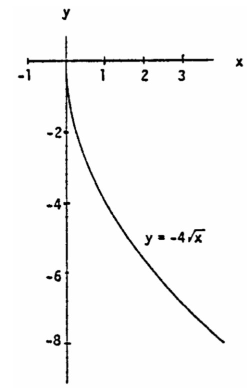
42. No symmetry
Dec: $-\infty < x \le 0$
Inc: nowhere



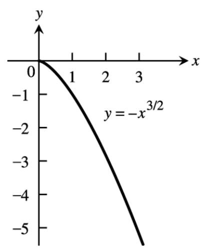
43. Symmetric about the origin
Dec: nowhere
Inc: $-\infty < x < \infty$



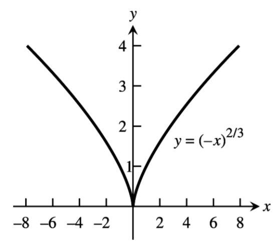
44. No symmetry
Dec: $0 \le x < \infty$
Inc: nowhere



45. No symmetry
Dec: $0 \le x < \infty$
Inc: nowhere



46. Symmetric about the y-axis
Dec: $-\infty < x \le 0$
Inc: $0 \le x < \infty$



47. Since a horizontal line not through the origin is symmetric with respect to the y-axis, but not with respect to the origin, the function is even.

48. $f(x) = x^{-5} = \frac{1}{x^5}$ and $f(-x) = (-x)^{-5} = \frac{1}{(-x)^5} = -\left(\frac{1}{x^5}\right) = -f(x)$. Thus the function is odd.

49. Since $f(x) = x^2 + 1 = (-x)^2 + 1 = f(-x)$. The function is even.

50. Since $[f(x) = x^2 + x] \neq [f(-x) = (-x)^2 - x]$ and $[f(x) = x^2 + x] \neq [-f(x) = -(x)^2 - x]$ the function is neither even nor odd.

51. Since $g(x) = x^3 + x$, $g(-x) = -x^3 - x = -(x^3 + x) = -g(x)$. So the function is odd.

52. $g(x) = x^4 + 3x^2 - 1 = (-x)^4 + 3(-x)^2 - 1 = g(-x)$, thus the function is even.

53. $g(x) = \frac{1}{x^2 - 1} = \frac{1}{(-x)^2 - 1} = g(-x)$. Thus the function is even.

54. $g(x) = \frac{x}{x^2 - 1}$; $g(-x) = -\frac{x}{x^2 - 1} = -g(x)$. So the function is odd.

55. $h(t) = \frac{1}{t-1}$; $h(-t) = \frac{1}{-t-1}$; $-h(t) = \frac{1}{1-t}$. Since $h(t) \neq -h(t)$ and $h(t) \neq h(-t)$, the function is neither even nor odd.

56. Since $|t^3| = |(-t)^3|, h(t) = h(-t)$ and the function is even.

57. $h(t) = 2t + 1$, $h(-t) = -2t + 1$. So $h(t) \neq h(-t)$. $-h(t) = -2t - 1$, so $h(t) \neq -h(t)$. The function is neither even nor odd.

58. $h(t) = 2|t| + 1$ and $h(-t) = 2|-t| + 1 = 2|t| + 1$. So $h(t) = h(-t)$ and the function is even.

59. $s = kt \Rightarrow 25 = k(75) \Rightarrow k = \frac{1}{3} \Rightarrow s = \frac{1}{3}t$; $60 = \frac{1}{3}t \Rightarrow t = 180$

60. $K = c\,v^2 \Rightarrow 12960 = c(18)^2 \Rightarrow c = 40 \Rightarrow K = 40v^2$; $K = 40(10)^2 = 4000$ joules

61. $r = \frac{k}{s} \Rightarrow 6 = \frac{k}{4} \Rightarrow k = 24 \Rightarrow r = \frac{24}{s}$; $10 = \frac{24}{s} \Rightarrow s = \frac{12}{5}$

62. $P = \frac{k}{V} \Rightarrow 14.7 = \frac{k}{1000} \Rightarrow k = 14700 \Rightarrow P = \frac{14700}{V}$; $23.4 = \frac{14700}{V} \Rightarrow V = \frac{24500}{39} \approx 628.2 \text{ in}^3$

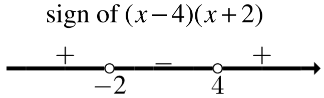
63. $V = f(x) = x(14 - 2x)(22 - 2x) = 4x^3 - 72x^2 + 308x$; $0 < x < 7$.

64. (a) Let h = height of the triangle. Since the triangle is isosceles, $\left(\overline{AB}\right)^2 + \left(\overline{AB}\right)^2 = 2^2 \Rightarrow \overline{AB} = \sqrt{2}$. So, $h^2 + 1^2 = \left(\sqrt{2}\right)^2 \Rightarrow h = 1 \Rightarrow B$ is at $(0, 1) \Rightarrow$ slope of $AB = -1 \Rightarrow$ The equation of AB is $y = f(x) = -x + 1$; $x \in [0, 1]$.
(b) $A(x) = 2xy = 2x(-x + 1) = -2x^2 + 2x$; $x \in [0, 1]$.

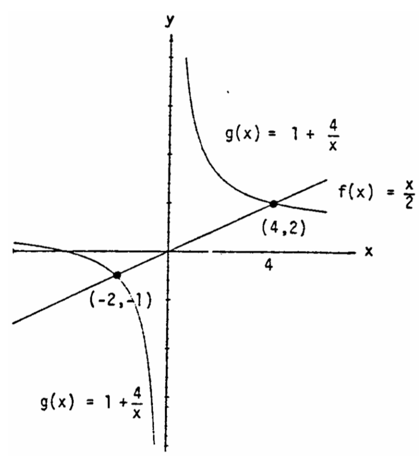
65. (a) Graph h because it is an even function and rises less rapidly than does Graph g.
(b) Graph f because it is an odd function.
(c) Graph g because it is an even function and rises more rapidly than does Graph h.

66. (a) Graph f because it is linear.
(b) Graph g because it contains (0, 1).
(c) Graph h because it is a nonlinear odd function.

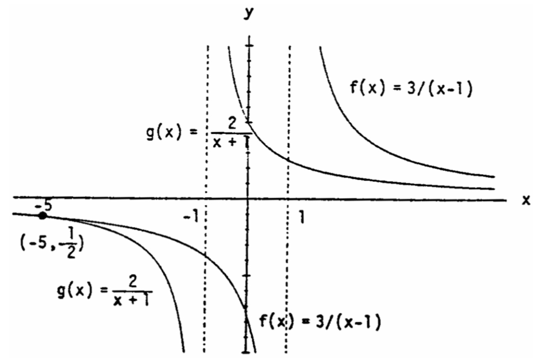
67. (a) From the graph, $\frac{x}{2} > 1 + \frac{4}{x} \Rightarrow x \in (-2, 0) \cup (4, \infty)$
(b) $\frac{x}{2} > 1 + \frac{4}{x} \Rightarrow \frac{x}{2} - 1 - \frac{4}{x} > 0$
$x > 0$: $\frac{x}{2} - 1 - \frac{4}{x} > 0 \Rightarrow \frac{x^2 - 2x - 8}{2x} > 0 \Rightarrow \frac{(x-4)(x+2)}{2x} > 0$
$\Rightarrow x > 4$ since x is positive;
$x < 0$: $\frac{x}{2} - 1 - \frac{4}{x} > 0 \Rightarrow \frac{x^2 - 2x - 8}{2x} < 0 \Rightarrow \frac{(x-4)(x+2)}{2x} < 0$
$\Rightarrow x < -2$ since x is negative;
sign of $(x - 4)(x + 2)$



Solution interval: $(-2, 0) \cup (4, \infty)$



68. (a) From the graph, $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow x \in (-\infty, -5) \cup (-1, 1)$



(b) Case $x < -1$: $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow \frac{3(x+1)}{x-1} > 2$
$\Rightarrow 3x+3 < 2x-2 \Rightarrow x < -5.$
Thus, $x \in (-\infty, -5)$ solves the inequality.

Case $-1 < x < 1$: $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow \frac{3(x+1)}{x-1} < 2$
$\Rightarrow 3x+3 > 2x-2 \Rightarrow x > -5$ which is true if $x > -1$. Thus, $x \in (-1, 1)$ solves the inequality.

Case $1 < x$: $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow 3x+3 < 2x-2 \Rightarrow x < -5$ which is never true if $1 < x$, so no solution here.

In conclusion, $x \in (-\infty, -5) \cup (-1, 1)$.

69. A curve symmetric about the x-axis will not pass the vertical line test because the points (x, y) and $(x, -y)$ lie on the same vertical line. The graph of the function $y = f(x) = 0$ is the x-axis, a horizontal line for which there is a single y-value, 0, for any x.

70. price $= 40 + 5x$, quantity $= 300 - 25x \Rightarrow R(x) = (40+5x)(300-25x)$

71. $x^2 + x^2 = h^2 \Rightarrow x = \frac{h}{\sqrt{2}} = \frac{\sqrt{2}h}{2}$; cost $= 5(2x) + 10h \Rightarrow C(h) = 10\left(\frac{\sqrt{2}h}{2}\right) + 10h = 5h\left(\sqrt{2}+2\right)$

72. (a) Note that 2 mi $= 10{,}560$ ft, so there are $\sqrt{800^2 + x^2}$ feet of river cable at \$180 per foot and $(10{,}560 - x)$ feet of land cable at \$100 per foot. The cost is $C(x) = 180\sqrt{800^2 + x^2} + 100(10{,}560 - x)$.

(b) $C(0) = \$1{,}200{,}000$
$C(500) \approx \$1{,}175{,}812$
$C(1000) \approx \$1{,}186{,}512$
$C(1500) \approx \$1{,}212{,}000$
$C(2000) \approx \$1{,}243{,}732$
$C(2500) \approx \$1{,}278{,}479$
$C(3000) \approx \$1{,}314{,}870$
Values beyond this are all larger. It would appear that the least expensive location is less than 2000 feet from the point P.

1.2 COMBINING FUNCTIONS; SHIFTING AND SCALING GRAPHS

1. $D_f: -\infty < x < \infty$, $D_g: x \ge 1 \Rightarrow D_{f+g} = D_{fg}: x \ge 1$. $R_f: -\infty < y < \infty$, $R_g: y \ge 0$, $R_{f+g}: y \ge 1$, $R_{fg}: y \ge 0$

2. $D_f: x+1 \ge 0 \Rightarrow x \ge -1$, $D_g: x-1 \ge 0 \Rightarrow x \ge 1$. Therefore $D_{f+g} = D_{fg}: x \ge 1$.
$R_f = R_g: y \ge 0$, $R_{f+g}: y \ge \sqrt{2}$, $R_{fg}: y \ge 0$

3. $D_f: -\infty < x < \infty$, $D_g: -\infty < x < \infty$, $D_{f/g}: -\infty < x < \infty$, $D_{g/f}: -\infty < x < \infty$, $R_f: y = 2$, $R_g: y \ge 1$, $R_{f/g}: 0 < y \le 2$, $R_{g/f}: \frac{1}{2} \le y < \infty$

4. $D_f: -\infty < x < \infty$, $D_g: x \ge 0$, $D_{f/g}: x \ge 0$, $D_{g/f}: x \ge 0$; $R_f: y = 1$, $R_g: y \ge 1$, $R_{f/g}: 0 < y \le 1$, $R_{g/f}: 1 \le y < \infty$

5. (a) 2 (b) 22 (c) x^2+2
(d) $(x+5)^2-3=x^2+10x+22$ (e) 5 (f) -2
(g) $x+10$ (h) $(x^2-3)^2-3=x^4-6x^2+6$

6. (a) $-\frac{1}{3}$ (b) 2 (c) $\frac{1}{x+1}-1=\frac{-x}{x+1}$
(d) $\frac{1}{x}$ (e) 0 (f) $\frac{3}{4}$
(g) $x-2$ (h) $\frac{1}{\frac{1}{x+1}+1}=\frac{1}{\frac{x+2}{x+1}}=\frac{x+1}{x+2}$

7. $(f\circ g\circ h)(x)=f(g(h(x)))=f(g(4-x))=f(3(4-x))=f(12-3x)=(12-3x)+1=13-3x$

8. $(f\circ g\circ h)(x)=f(g(h(x)))=f(g(x^2))=f(2(x^2)-1)=f(2x^2-1)=3(2x^2-1)+4=6x^2+1$

9. $(f\circ g\circ h)(x)=f(g(h(x)))=f\left(g\left(\frac{1}{x}\right)\right)=f\left(\frac{1}{\frac{1}{x}+4}\right)=f\left(\frac{x}{1+4x}\right)=\sqrt{\frac{x}{1+4x}+1}=\sqrt{\frac{5x+1}{1+4x}}$

10. $(f\circ g\circ h)(x)=f(g(h(x)))=f\left(g\left(\sqrt{2-x}\right)\right)=f\left(\frac{\left(\sqrt{2-x}\right)^2}{\left(\sqrt{2-x}\right)^2+1}\right)=f\left(\frac{2-x}{3-x}\right)=\frac{\frac{2-x}{3-x}+2}{3-\frac{2-x}{3-x}}=\frac{8-3x}{7-2x}$

11. (a) $(f\circ g)(x)$ (b) $(j\circ g)(x)$ (c) $(g\circ g)(x)$
(d) $(j\circ j)(x)$ (e) $(g\circ h\circ f)(x)$ (f) $(h\circ j\circ f)(x)$

12. (a) $(f\circ j)(x)$ (b) $(g\circ h)(x)$ (c) $(h\circ h)(x)$
(d) $(f\circ f)(x)$ (e) $(j\circ g\circ f)(x)$ (f) $(g\circ f\circ h)(x)$

13.

	$g(x)$	$f(x)$	$(f\circ g)(x)$
(a)	$x-7$	$\sqrt{x}$	$\sqrt{x-7}$
(b)	$x+2$	$3x$	$3(x+2)=3x+6$
(c)	x^2	$\sqrt{x-5}$	$\sqrt{x^2-5}$
(d)	$\frac{x}{x-1}$	$\frac{x}{x-1}$	$\frac{\frac{x}{x-1}}{\frac{x}{x-1}-1}=\frac{x}{x-(x-1)}=x$
(e)	$\frac{1}{x-1}$	$1+\frac{1}{x}$	x
(f)	$\frac{1}{x}$	$\frac{1}{x}$	x

14. (a) $(f\circ g)(x)=|g(x)|=\frac{1}{|x-1|}$.
(b) $(f\circ g)(x)=\frac{g(x)-1}{g(x)}=\frac{x}{x+1}\Rightarrow 1-\frac{1}{g(x)}=\frac{x}{x+1}\Rightarrow 1-\frac{x}{x+1}=\frac{1}{g(x)}\Rightarrow\frac{1}{x+1}=\frac{1}{g(x)}$, so $g(x)=x+1$.
(c) Since $(f\circ g)(x)=\sqrt{g(x)}=|x|$, $g(x)=x^2$.
(d) Since $(f\circ g)(x)=f\left(\sqrt{x}\right)=|x|$, $f(x)=x^2$. (Note that the domain of the composite is $[0,\infty)$.)

The completed table is shown. Note that the absolute value sign in part (d) is optional.

$g(x)$	$f(x)$	$(f \circ g)(x)$
$\frac{1}{x-1}$	$\lvert x \rvert$	$\frac{1}{\lvert x-1 \rvert}$
$x+1$	$\frac{x-1}{x}$	$\frac{x}{x+1}$
x^2	$\sqrt{x}$	$\lvert x \rvert$
$\sqrt{x}$	x^2	$\lvert x \rvert$

15. (a) $f(g(-1)) = f(1) = 1$ (b) $g(f(0)) = g(-2) = 2$ (c) $f(f(-1)) = f(0) = -2$
(d) $g(g(2)) = g(0) = 0$ (e) $g(f(-2)) = g(1) = -1$ (f) $f(g(1)) = f(-1) = 0$

16. (a) $f(g(0)) = f(-1) = 2-(-1) = 3$, where $g(0) = 0-1 = -1$
(b) $g(f(3)) = g(-1) = -(-1) = 1$, where $f(3) = 2-3 = -1$
(c) $g(g(-1)) = g(1) = 1-1 = 0$, where $g(-1) = -(-1) = 1$
(d) $f(f(2)) = f(0) = 2-0 = 2$, where $f(2) = 2-2 = 0$
(e) $g(f(0)) = g(2) = 2-1 = 1$, where $f(0) = 2-0 = 2$
(f) $f\left(g\left(\frac{1}{2}\right)\right) = f\left(-\frac{1}{2}\right) = 2-\left(-\frac{1}{2}\right) = \frac{5}{2}$, where $g\left(\frac{1}{2}\right) = \frac{1}{2}-1 = -\frac{1}{2}$

17. (a) $(f \circ g)(x) = f(g(x)) = \sqrt{\frac{1}{x}+1} = \sqrt{\frac{1+x}{x}}$
$(g \circ f)(x) = g(f(x)) = \frac{1}{\sqrt{x+1}}$
(b) Domain $(f \circ g)$: $(-\infty, -1] \cup (0, \infty)$, domain $(g \circ f)$: $(-1, \infty)$
(c) Range $(f \circ g)$: $(1, \infty)$, range $(g \circ f)$: $(0, \infty)$

18. (a) $(f \circ g)(x) = f(g(x)) = 1 - 2\sqrt{x} + x$
$(g \circ f)(x) = g(f(x)) = 1 - \lvert x \rvert$
(b) Domain $(f \circ g)$: $[0, \infty)$, domain $(g \circ f)$: $(-\infty, \infty)$
(c) Range $(f \circ g)$: $(0, \infty)$, range $(g \circ f)$: $(-\infty, 1]$

19. $(f \circ g)(x) = x \Rightarrow f(g(x)) = x \Rightarrow \frac{g(x)}{g(x)-2} = x \Rightarrow g(x) = (g(x)-2)x = x \cdot g(x) - 2x$
$\Rightarrow g(x) - x \cdot g(x) = -2x \Rightarrow g(x) = -\frac{2x}{1-x} = \frac{2x}{x-1}$

20. $(f \circ g)(x) = x+2 \Rightarrow f(g(x)) = x+2 \Rightarrow 2(g(x))^3 - 4 = x+2 \Rightarrow (g(x))^3 = \frac{x+6}{2} \Rightarrow g(x) = \sqrt[3]{\frac{x+6}{2}}$

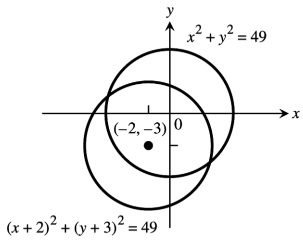
21. (a) $y = -(x+7)^2$ (b) $y = -(x-4)^2$

22. (a) $y = x^2 + 3$ (b) $y = x^2 - 5$

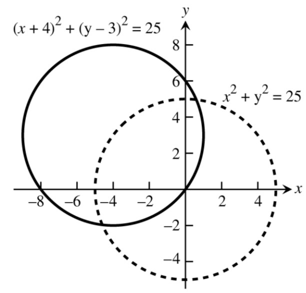
23. (a) Position 4 (b) Position 1 (c) Position 2 (d) Position 3

24. (a) $y = -(x-1)^2 + 4$ (b) $y = -(x+2)^2 + 3$ (c) $y = -(x+4)^2 - 1$ (d) $y = -(x-2)^2$

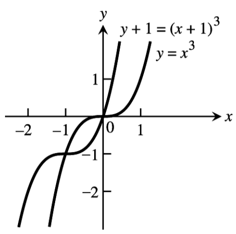
25.



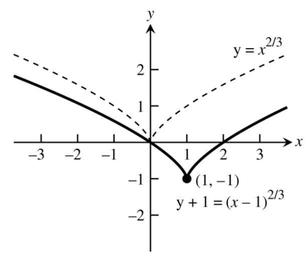
26.



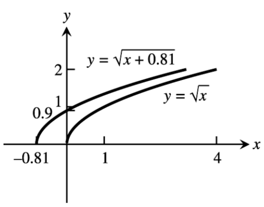
27.



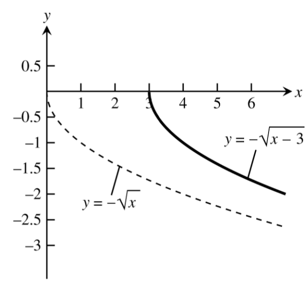
28.



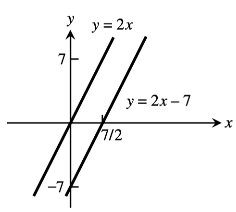
29.



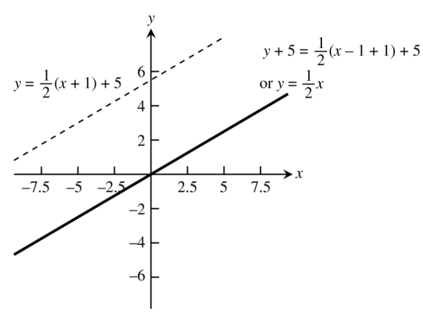
30.



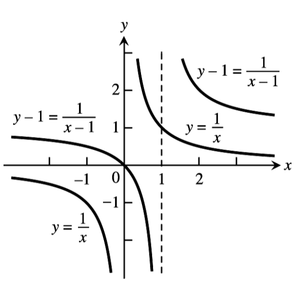
31.



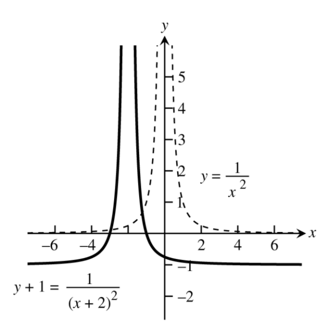
32.



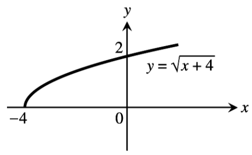
33.



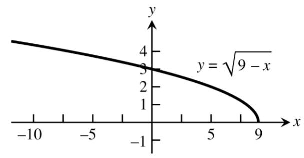
34.



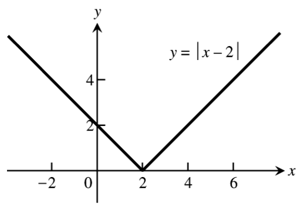
35.



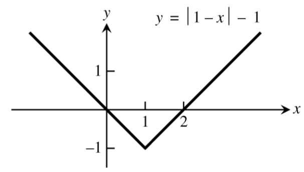
36.



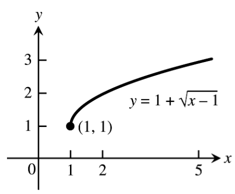
37.



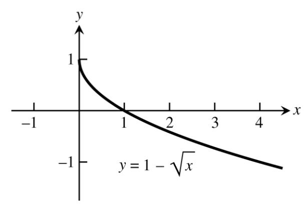
38.



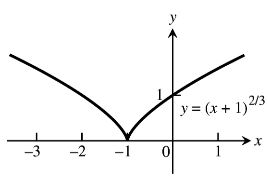
39.



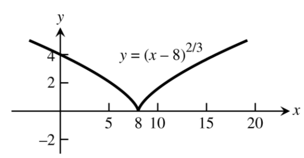
40.



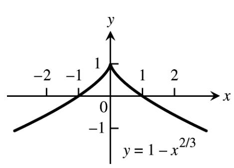
41.



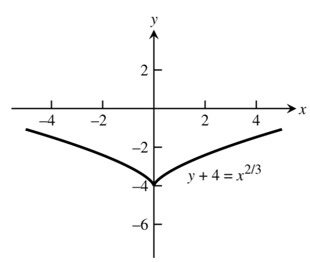
42.



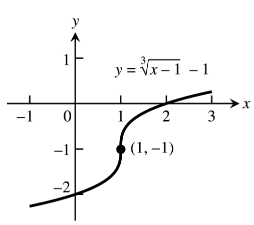
43.



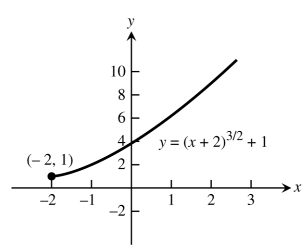
44.



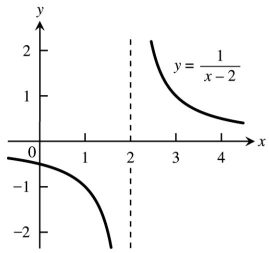
45.



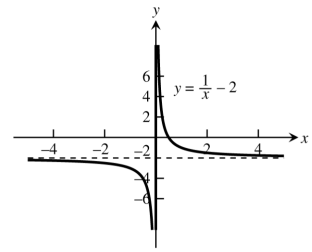
46.



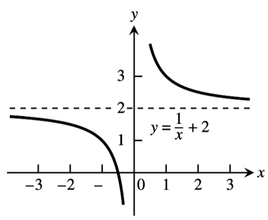
47.



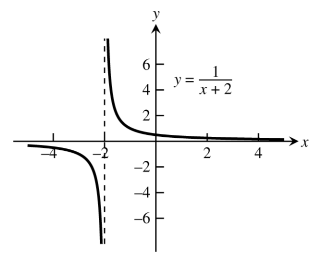
48.



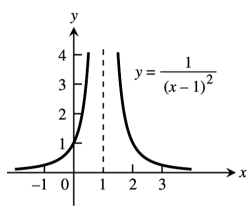
49.



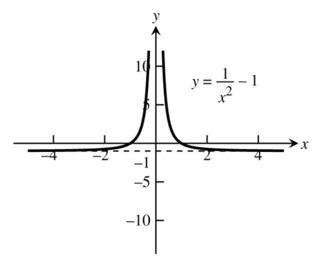
50.



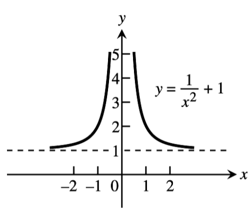
51.



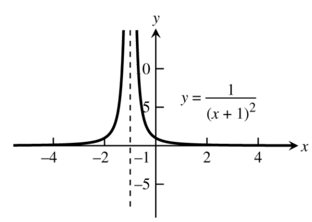
52.



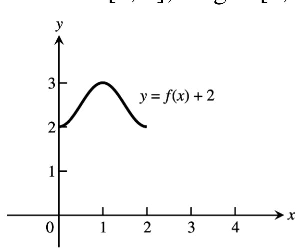
53.



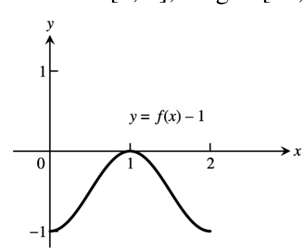
54.



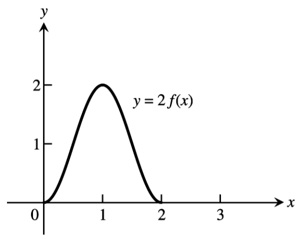
55. (a) domain: [0, 2]; range: [2, 3]



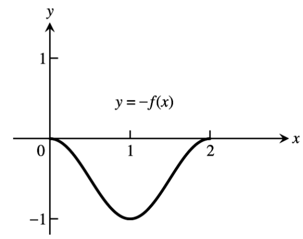
(b) domain: [0, 2]; range: [–1, 0]



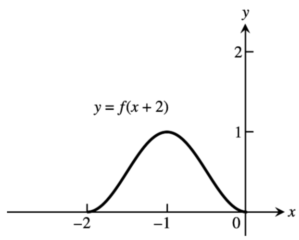
(c) domain: [0, 2]; range: [0, 2]



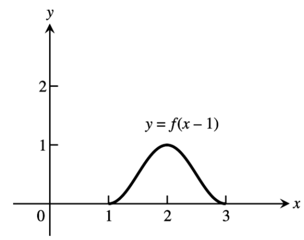
(d) domain: [0, 2]; range: [–1, 0]



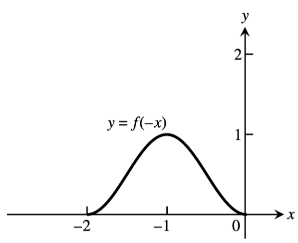
(e) domain: [–2, 0]; range: [0, 1]



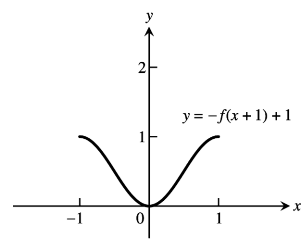
(f) domain: [1, 3]; range: [0,1]



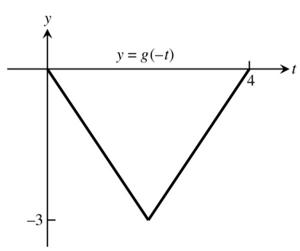
(g) domain: [–2, 0]; range: [0, 1]



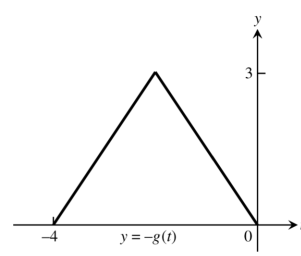
(h) domain: [–1, 1]; range: [0, 1]



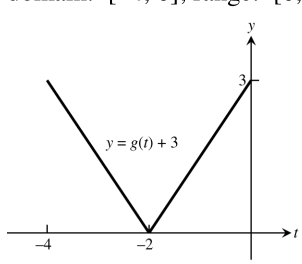
56. (a) domain: [0, 4]; range: [–3, 0]



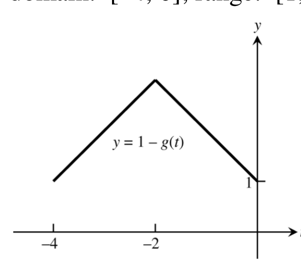
(b) domain: [–4, 0]; range: [0, 3]



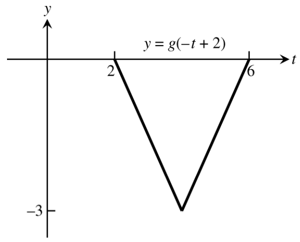
(c) domain: [–4, 0]; range: [0, 3]



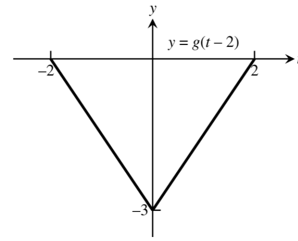
(d) domain: [–4, 0]; range: [1, 4]



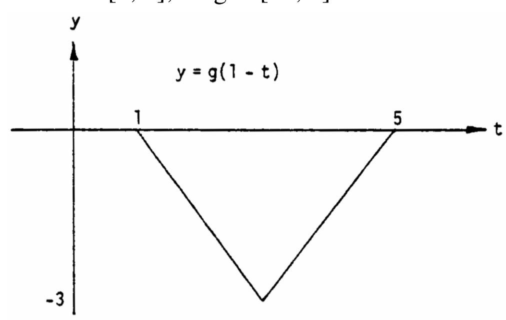
(e) domain: [2, 4]; range: [–3, 0]



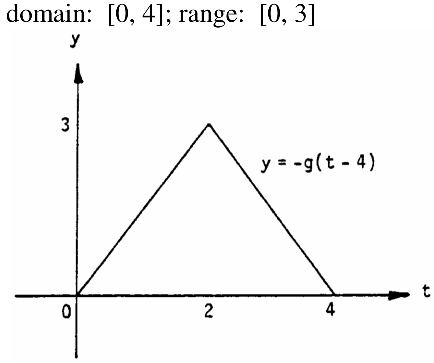
(f) domain: [–2, 2]; range: [–3, 0]



(g) domain: [1, 5]; range: [–3, 0]



(h) domain: [0, 4]; range: [0, 3]



57. $y = 3x^2 - 3$

58. $y = (2x)^2 - 1 = 4x^2 - 1$

59. $y = \frac{1}{2}\left(1 + \frac{1}{x^2}\right) = \frac{1}{2} + \frac{1}{2x^2}$

60. $y = 1 + \frac{1}{(x/3)^2} = 1 + \frac{9}{x^2}$

61. $y = \sqrt{4x+1}$

62. $y = 3\sqrt{x+1}$

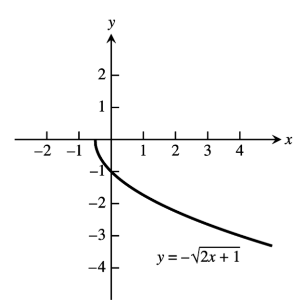
63. $y = \sqrt{4 - \left(\frac{x}{2}\right)^2} = \frac{1}{2}\sqrt{16 - x^2}$

64. $y = \frac{1}{3}\sqrt{4 - x^2}$

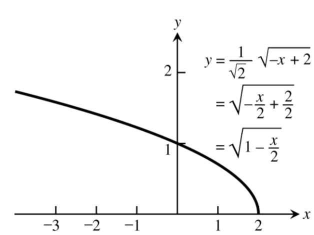
65. $y = 1 - (3x)^3 = 1 - 27x^3$

66. $y = 1 - \left(\frac{x}{2}\right)^3 = 1 - \frac{x^3}{8}$

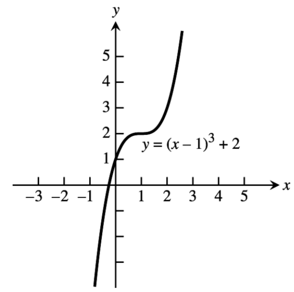
67. Let $y = -\sqrt{2x+1} = f(x)$ and let $g(x) = x^{1/2}$, $h(x) = \left(x + \frac{1}{2}\right)^{1/2}$, $i(x) = \sqrt{2}\left(x + \frac{1}{2}\right)^{1/2}$, and $j(x) = -\left[\sqrt{2}\left(x + \frac{1}{2}\right)^{1/2}\right] = f(x)$. The graph of $h(x)$ is the graph of $g(x)$ shifted left $\frac{1}{2}$ unit; the graph of $i(x)$ is the graph of $h(x)$ stretched vertically by a factor of $\sqrt{2}$; and the graph of $j(x) = f(x)$ is the graph of $i(x)$ reflected across the x-axis.



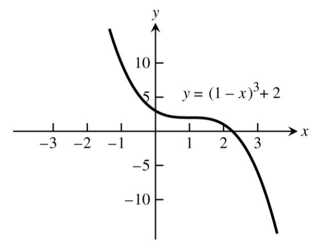
68. Let $y = \sqrt{1 - \frac{x}{2}} = f(x)$. Let $g(x) = (-x)^{1/2}$, $h(x) = (-x+2)^{1/2}$, and $i(x) = \frac{1}{\sqrt{2}}(-x+2)^{1/2} = \sqrt{1 - \frac{x}{2}} = f(x)$. The graph of $g(x)$ is the graph of $y = \sqrt{x}$ reflected across the x-axis. The graph of $h(x)$ is the graph of $g(x)$ shifted right two units. And the graph of $i(x)$ is the graph of $h(x)$ compressed vertically by a factor of $\sqrt{2}$.



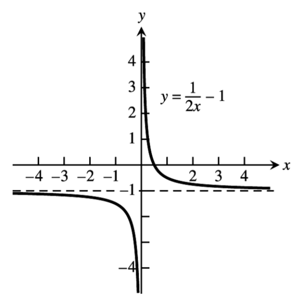
69. $y = f(x) = x^3$. Shift $f(x)$ one unit right followed by a shift two units up to get $g(x) = (x-1)^3 + 2$.



70. $y = (1-x)^3 + 2 = -[(x-1)^3 + (-2)] = f(x)$.
Let $g(x) = x^3$, $h(x) = (x-1)^3$, $i(x) = (x-1)^3 + (-2)$, and $j(x) = -[(x-1)^3 + (-2)]$. The graph of $h(x)$ is the graph of $g(x)$ shifted right one unit; the graph of $i(x)$ is the graph of $h(x)$ shifted down two units; and the graph of $f(x)$ is the graph of $i(x)$ reflected across the x-axis.

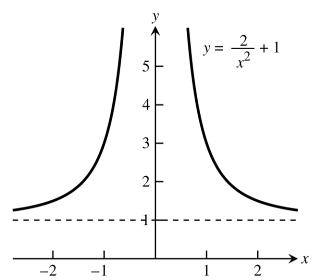


71. Compress the graph of $f(x) = \frac{1}{x}$ horizontally by a factor of 2 to get $g(x) = \frac{1}{2x}$. Then shift $g(x)$ vertically down 1 unit to get $h(x) = \frac{1}{2x} - 1$.

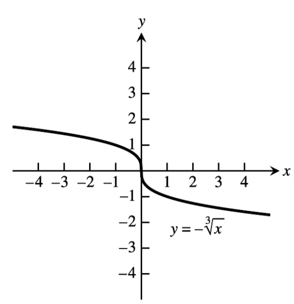


72. Let $f(x) = \frac{1}{x^2}$ and $g(x) = \frac{2}{x^2} + 1 = \frac{1}{\left(\frac{x^2}{2}\right)} + 1$

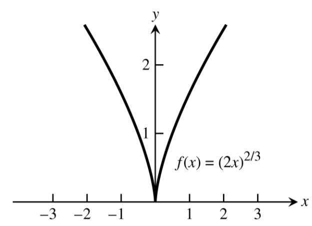
$= \frac{1}{\left(x/\sqrt{2}\right)^2} + 1 = \frac{1}{\left[\left(1/\sqrt{2}\right)x\right]^2} + 1.$ Since $\sqrt{2} \approx 1.4$, we see that the graph of $f(x)$ stretched horizontally by a factor of 1.4 and shifted up 1 unit is the graph of $g(x)$.



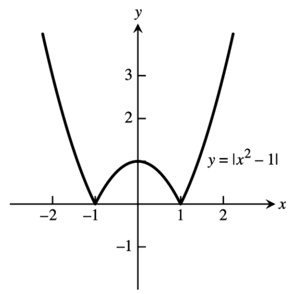
73. Reflect the graph of $y = f(x) = \sqrt[3]{x}$ across the x-axis to get $g(x) = -\sqrt[3]{x}$.



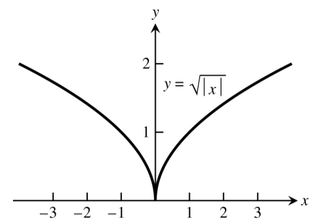
74. $y = f(x) = (-2x)^{2/3} = [(-1)(2)x]^{2/3} = (-1)^{2/3}(2x)^{2/3}$ $= (2x)^{2/3}$. So the graph of $f(x)$ is the graph of $g(x) = x^{2/3}$ compressed horizontally by a factor of 2.



75.



76.



77. (a) $(fg)(-x) = f(-x)g(-x) = f(x)(-g(x)) = -(fg)(x)$, odd

(b) $\left(\frac{f}{g}\right)(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{-g(x)} = -\left(\frac{f}{g}\right)(x)$, odd

(c) $\left(\frac{g}{f}\right)(-x) = \frac{g(-x)}{f(-x)} = \frac{-g(x)}{f(x)} = -\left(\frac{g}{f}\right)(x)$, odd

(d) $f^2(-x) = f(-x)f(-x) = f(x)f(x) = f^2(x)$, even

(e) $g^2(-x) = (g(-x))^2 = (-g(x))^2 = g^2(x)$, even

(f) $(f \circ g)(-x) = f(g(-x)) = f(-g(x)) = f(g(x)) = (f \circ g)(x)$, even

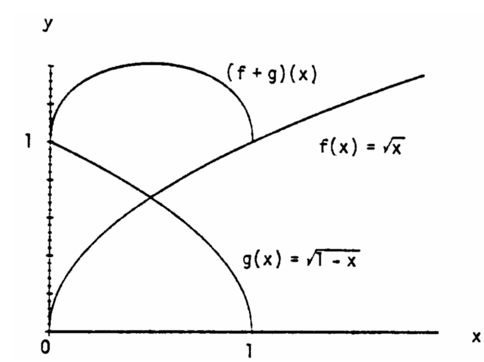
(g) $(g \circ f)(-x) = g(f(-x)) = g(f(x)) = (g \circ f)(x)$, even

(h) $(f \circ f)(-x) = f(f(-x)) = f(f(x)) = (f \circ f)(x)$, even

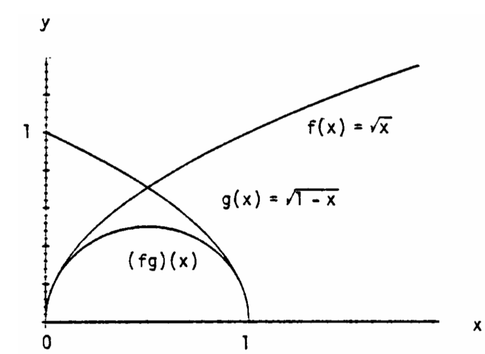
(i) $(g \circ g)(-x) = g(g(-x)) = g(-g(x)) = -g(g(x)) = -(g \circ g)(x)$, odd

78. Yes, $f(x) = 0$ is both even and odd since $f(-x) = 0 = f(x)$ and $f(-x) = 0 = -f(x)$.

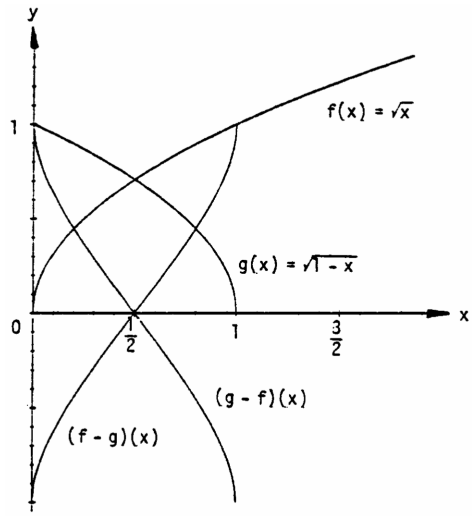
79. (a)



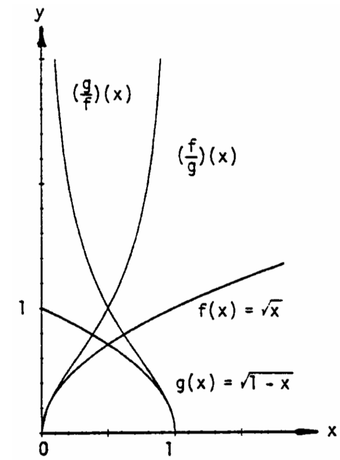
(b)



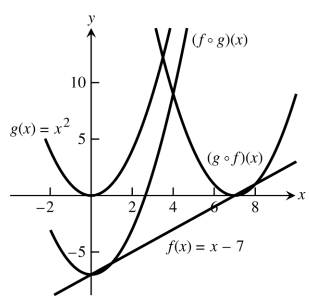
(c)



(d)



80.



1.3 TRIGONOMETRIC FUNCTIONS

1. (a) $s = r\theta = (10)\left(\frac{4\pi}{5}\right) = 8\pi$ m (b) $s = r\theta = (10)(110°)\left(\frac{\pi}{180°}\right) = \frac{110\pi}{18} = \frac{55\pi}{9}$ m

2. $\theta = \frac{s}{r} = \frac{10\pi}{8} = \frac{5\pi}{4}$ radians and $\frac{5\pi}{4}\left(\frac{180°}{\pi}\right) = 225°$

3. $\theta = 80° \Rightarrow \theta = 80°\left(\frac{\pi}{180°}\right) = \frac{4\pi}{9} \Rightarrow s = (6)\left(\frac{4\pi}{9}\right) = 8.4$ in. (since the diameter $= 12$ in. $\Rightarrow$ radius $= 6$ in.)

4. $d = 1$ meter $\Rightarrow r = 50$ cm $\Rightarrow \theta = \frac{s}{r} = \frac{30}{50} = 0.6$ rad or $0.6\left(\frac{180°}{\pi}\right) \approx 34°$

5.

θ	$-\pi$	$-\frac{2\pi}{3}$	0	$\frac{\pi}{2}$	$\frac{3\pi}{4}$
$\sin\theta$	0	$-\frac{\sqrt{3}}{2}$	0	1	$\frac{1}{\sqrt{2}}$
$\cos\theta$	–1	$-\frac{1}{2}$	1	0	$-\frac{1}{\sqrt{2}}$
$\tan\theta$	0	$\sqrt{3}$	0	und.	–1
$\cot\theta$	und.	$\frac{1}{\sqrt{3}}$	und.	0	–1
$\sec\theta$	–1	–2	1	und.	$-\sqrt{2}$
$\csc\theta$	und.	$-\frac{2}{\sqrt{3}}$	und.	1	$\sqrt{2}$

6.

θ	$-\frac{3\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{5\pi}{6}$
$\sin\theta$	1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\cos\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$
$\tan\theta$	und.	$-\sqrt{3}$	$-\frac{1}{\sqrt{3}}$	1	$-\frac{1}{\sqrt{3}}$
$\cot\theta$	0	$-\frac{1}{\sqrt{3}}$	$-\sqrt{3}$	1	$-\sqrt{3}$
$\sec\theta$	und.	2	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	$-\frac{2}{\sqrt{3}}$
$\csc\theta$	1	$-\frac{2}{\sqrt{3}}$	–2	$\sqrt{2}$	2

7. $\cos x = -\frac{4}{5}, \tan x = -\frac{3}{4}$

8. $\sin x = \frac{2}{\sqrt{5}}, \ \cos x = \frac{1}{\sqrt{5}}$

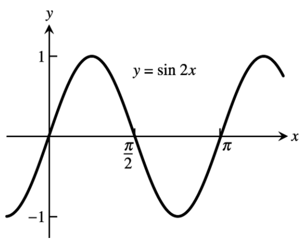
9. $\sin x = -\frac{\sqrt{8}}{3}, \tan x = -\sqrt{8}$

10. $\sin x = \frac{12}{13}, \tan x = -\frac{12}{5}$

11. $\sin x = -\frac{1}{\sqrt{5}}, \cos x = -\frac{2}{\sqrt{5}}$

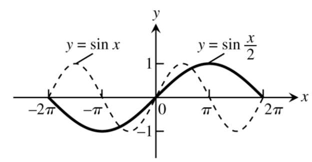
12. $\cos x = -\frac{\sqrt{3}}{2}, \tan x = \frac{1}{\sqrt{3}}$

13.



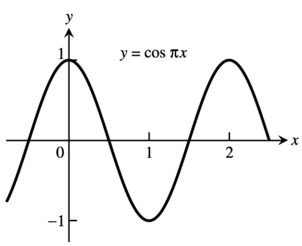
period $= \pi$

14.



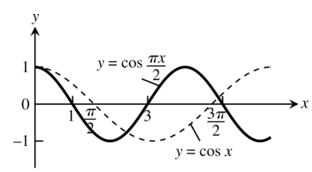
period $= 4\pi$

15.



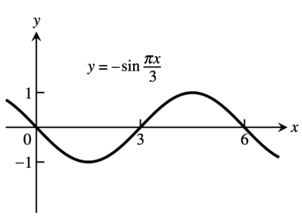
period $= 2$

16.



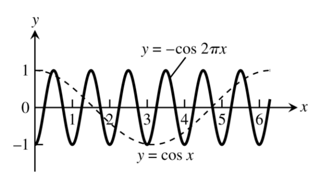
period $= 4$

17.



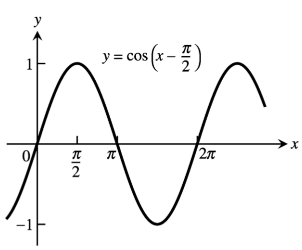
period $= 6$

18.



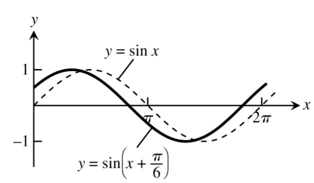
period $= 1$

19.



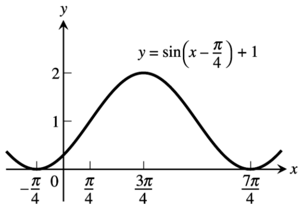
period $= 2\pi$

20.



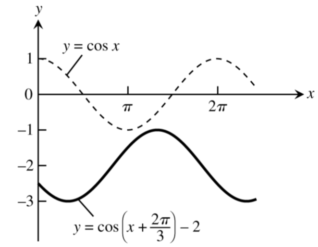
period $= 2\pi$

21.



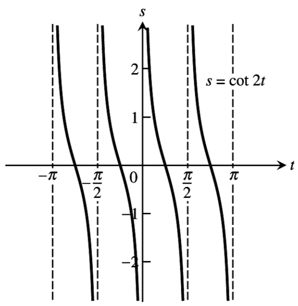
period $= 2\pi$

22.

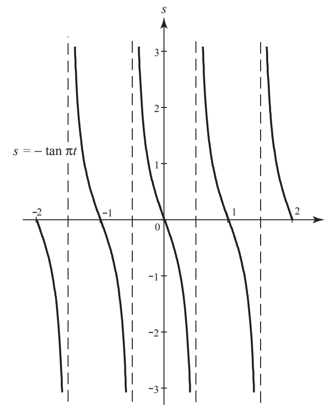


period $= 2\pi$

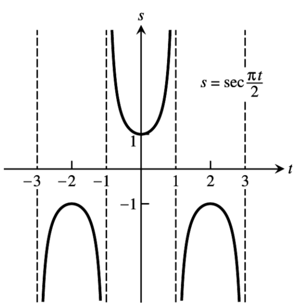
23. period $= \frac{\pi}{2}$, symmetric about the origin



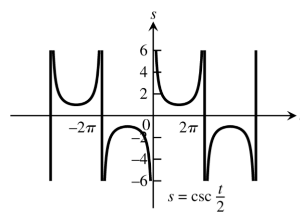
24. period $= 1$, symmetric about the origin



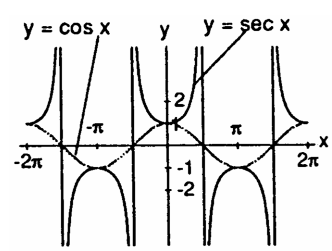
25. period $= 4$, symmetric about the s-axis



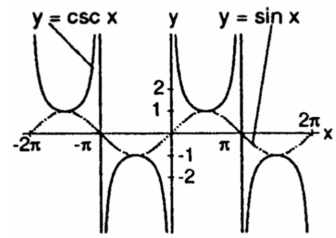
26. period $= 4\pi$, symmetric about the origin



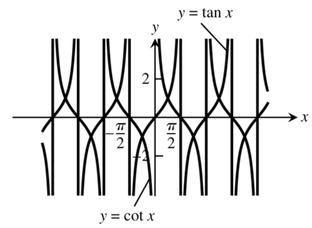
27. (a) Cos x and sec x are positive for x in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$; and cos x and sec x are negative for x in the intervals $\left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right)$ and $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$. Sec x is undefined when cos x is 0. The range of sec x is $(-\infty, -1] \cup [1, \infty)$; the range of cos x is $[-1, 1]$.



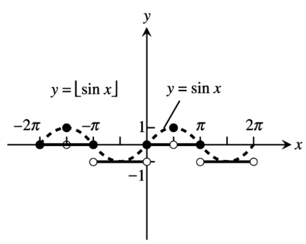
(b) Sin x and csc x are positive for x in the intervals $\left(-\frac{3\pi}{2}, -\pi\right)$ and $(0, \pi)$; and sin x and csc x are negative for x in the intervals $(-\pi, 0)$ and $\left(\pi, \frac{3\pi}{2}\right)$. Csc x is undefined when sin x is 0. The range of csc x is $(-\infty, -1] \cup [1, \infty)$; the range of sin x is $[-1, 1]$.



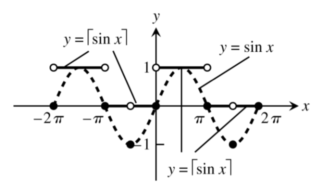
28. Since $\cot x = \frac{1}{\tan x}$, cot x is undefined when $\tan x = 0$ and is zero when tan x is undefined. As tan x approaches zero through positive values, cot x approaches infinity. Also, cot x approaches negative infinity as tan x approaches zero through negative values.



29. $D: -\infty < x < \infty;\ R:\ y = -1, 0, 1$



30. $D: -\infty < x < \infty;\ R:\ y = -1, 0, 1$



31. $\cos\left(x - \frac{\pi}{2}\right) = \cos x \cos\left(-\frac{\pi}{2}\right) - \sin x \sin\left(-\frac{\pi}{2}\right) = (\cos x)(0) - (\sin x)(-1) = \sin x$

32. $\cos\left(x + \frac{\pi}{2}\right) = \cos x \cos\left(\frac{\pi}{2}\right) - \sin x \sin\left(\frac{\pi}{2}\right) = (\cos x)(0) - (\sin x)(1) = -\sin x$

33. $\sin\left(x + \frac{\pi}{2}\right) = \sin x \cos\left(\frac{\pi}{2}\right) + \cos x \sin\left(\frac{\pi}{2}\right) = (\sin x)(0) + (\cos x)(1) = \cos x$

34. $\sin\left(x - \frac{\pi}{2}\right) = \sin x \cos\left(-\frac{\pi}{2}\right) + \cos x \sin\left(-\frac{\pi}{2}\right) = (\sin x)(0) + (\cos x)(-1) = -\cos x$

35. $\cos(A - B) = \cos(A + (-B)) = \cos A \cos(-B) - \sin A \sin(-B) = \cos A \cos B - \sin A(-\sin B)$
$= \cos A \cos B + \sin A \sin B$

36. $\sin(A - B) = \sin(A + (-B)) = \sin A \cos(-B) + \cos A \sin(-B) = \sin A \cos B + \cos A(-\sin B)$
$= \sin A \cos B - \cos A \sin B$

37. If $B = A,\ A - B = 0 \Rightarrow \cos(A - B) = \cos 0 = 1$. Also $\cos(A - B) = \cos(A - A) = \cos A \cos A + \sin A \sin A$ $= \cos^2 A + \sin^2 A$. Therefore, $\cos^2 A + \sin^2 A = 1$.

38. If $B = 2\pi$, then $\cos(A + 2\pi) = \cos A \cos 2\pi - \sin A \sin 2\pi = (\cos A)(1) - (\sin A)(0) = \cos A$ and $\sin(A + 2\pi) = \sin A \cos 2\pi + \cos A \sin 2\pi = (\sin A)(1) + (\cos A)(0) = \sin A$. The result agrees with the fact that the cosine and sine functions have period 2π.

39. $\cos(\pi + x) = \cos \pi \cos x - \sin \pi \sin x = (-1)(\cos x) - (0)(\sin x) = -\cos x$

40. $\sin(2\pi - x) = \sin 2\pi \cos(-x) + \cos(2\pi)\sin(-x) = (0)(\cos(-x)) + (1)(\sin(-x)) = -\sin x$

41. $\sin\left(\frac{3\pi}{2} - x\right) = \sin\left(\frac{3\pi}{2}\right)\cos(-x) + \cos\left(\frac{3\pi}{2}\right)\sin(-x) = (-1)(\cos x) + (0)(\sin(-x)) = -\cos x$

42. $\cos\left(\frac{3\pi}{2} + x\right) = \cos\left(\frac{3\pi}{2}\right)\cos x - \sin\left(\frac{3\pi}{2}\right)\sin x = (0)(\cos x) - (-1)(\sin x) = \sin x$

43. $\sin\frac{7\pi}{12} = \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{3} + \cos\frac{\pi}{4}\sin\frac{\pi}{3} = \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{6} + \sqrt{2}}{4}$

44. $\cos\frac{11\pi}{12} = \cos\left(\frac{\pi}{4} + \frac{2\pi}{3}\right) = \cos\frac{\pi}{4}\cos\frac{2\pi}{3} - \sin\frac{\pi}{4}\sin\frac{2\pi}{3} = \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{2} + \sqrt{6}}{4}$

45. $\cos\frac{\pi}{12} = \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \cos\frac{\pi}{3}\cos\left(-\frac{\pi}{4}\right) - \sin\frac{\pi}{3}\sin\left(-\frac{\pi}{4}\right) = \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) = \frac{1+\sqrt{3}}{2\sqrt{2}}$

46. $\sin\frac{5\pi}{12} = \sin\left(\frac{2\pi}{3} - \frac{\pi}{4}\right) = \sin\left(\frac{2\pi}{3}\right)\cos\left(-\frac{\pi}{4}\right) + \cos\left(\frac{2\pi}{3}\right)\sin\left(-\frac{\pi}{4}\right) = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) = \frac{1+\sqrt{3}}{2\sqrt{2}}$

47. $\cos^2\frac{\pi}{8} = \frac{1 + \cos\left(\frac{2\pi}{8}\right)}{2} = \frac{1 + \frac{\sqrt{2}}{2}}{2} = \frac{2 + \sqrt{2}}{4}$

48. $\cos^2\frac{5\pi}{12} = \frac{1 + \cos\left(\frac{10\pi}{12}\right)}{2} = \frac{1 + \left(-\frac{\sqrt{3}}{2}\right)}{2} = \frac{2 - \sqrt{3}}{4}$

49. $\sin^2\frac{\pi}{12} = \frac{1 - \cos\left(\frac{2\pi}{12}\right)}{2} = \frac{1 - \frac{\sqrt{3}}{2}}{2} = \frac{2 - \sqrt{3}}{4}$

50. $\sin^2\frac{3\pi}{8} = \frac{1 - \cos\left(\frac{6\pi}{8}\right)}{2} = \frac{1 - \left(-\frac{\sqrt{2}}{2}\right)}{2} = \frac{2 + \sqrt{2}}{4}$

51. $\sin^2\theta = \frac{3}{4} \Rightarrow \sin\theta = \pm\frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

52. $\sin^2\theta = \cos^2\theta \Rightarrow \frac{\sin^2\theta}{\cos^2\theta} = \frac{\cos^2\theta}{\cos^2\theta} \Rightarrow \tan^2\theta = 1 \Rightarrow \tan\theta = \pm 1 \Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

53. $\sin 2\theta - \cos\theta = 0 \Rightarrow 2\sin\theta\cos\theta - \cos\theta = 0 \Rightarrow \cos\theta(2\sin\theta - 1) = 0 \Rightarrow \cos\theta = 0$ or $2\sin\theta - 1 = 0$
$\Rightarrow \cos\theta = 0$ or $\sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$, or $\theta = \frac{\pi}{6}, \frac{5\pi}{6} \Rightarrow \theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

54. $\cos 2\theta + \cos\theta = 0 \Rightarrow 2\cos^2\theta - 1 + \cos\theta = 0 \Rightarrow 2\cos^2\theta + \cos\theta - 1 = 0 \Rightarrow (\cos\theta + 1)(2\cos\theta - 1) = 0$
$\Rightarrow \cos\theta + 1 = 0$ or $2\cos\theta - 1 = 0 \Rightarrow \cos\theta = -1$ or $\cos\theta = \frac{1}{2} \Rightarrow \theta = \pi$ or $\theta = \frac{\pi}{3}, \frac{5\pi}{3} \Rightarrow \theta = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

55. $\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A\cos B + \cos A\cos B}{\cos A\cos B - \sin A\sin B} = \frac{\frac{\sin A\cos B}{\cos A\cos B} + \frac{\cos A\sin B}{\cos A\cos B}}{\frac{\cos A\cos B}{\cos A\cos B} - \frac{\sin A\sin B}{\cos A\cos B}} = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

56. $\tan(A-B) = \frac{\sin(A-B)}{\cos(A-B)} = \frac{\sin A\cos B - \cos A\cos B}{\cos A\cos B + \sin A\sin B} = \frac{\frac{\sin A\cos B}{\cos A\cos B} - \frac{\cos A\sin B}{\cos A\cos B}}{\frac{\cos A\cos B}{\cos A\cos B} + \frac{\sin A\sin B}{\cos A\cos B}} = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

57. According to the figure in the text, we have the following: By the law of cosines, $c^2 = a^2 + b^2 - 2ab\cos\theta$ $= 1^2 + 1^2 - 2\cos(A-B) = 2 - 2\cos(A-B)$. By distance formula, $c^2 = (\cos A - \cos B)^2 + (\sin A - \sin B)^2$ $= \cos^2 A - 2\cos A\cos B + \cos^2 B + \sin^2 A - 2\sin A\sin B + \sin^2 B = 2 - 2(\cos A\cos B + \sin A\sin B)$. Thus $c^2 = 2 - 2\cos(A-B) = 2 - 2(\cos A\cos B + \sin A\sin B) \Rightarrow \cos(A-B) = \cos A\cos B + \sin A\sin B$.

58. (a) $\cos(A-B)=\cos A\cos B+\sin A\sin B$
$\sin\theta=\cos\left(\frac{\pi}{2}-\theta\right)$ and $\cos\theta=\sin\left(\frac{\pi}{2}-\theta\right)$
Let $\theta=A+B$
$\sin(A+B)=\cos\left[\frac{\pi}{2}-(A+B)\right]=\cos\left[\left(\frac{\pi}{2}-A\right)-B\right]=\cos\left(\frac{\pi}{2}-A\right)\cos B+\sin\left(\frac{\pi}{2}-A\right)\sin B$
$=\sin A\cos B+\cos A\sin B$

(b) $\cos(A-B)=\cos A\cos B+\sin A\sin B$
$\cos(A-(-B))=\cos A\cos(-B)+\sin A\sin(-B)$
$\Rightarrow \cos(A+B)=\cos A\cos(-B)+\sin A\sin(-B)=\cos A\cos B+\sin A(-\sin B)=\cos A\cos B-\sin A\sin B$
Because the cosine function is even and the sine functions is odd.

59. $c^2=a^2+b^2-2ab\cos C=2^2+3^2-2(2)(3)\cos(60°)=4+9-12\cos(60°)=13-12\left(\frac{1}{2}\right)=7.$
Thus, $c=\sqrt{7}\approx 2.65$.

60. $c^2=a^2+b^2-2ab\cos C=2^2+3^2-2(2)(3)\cos(40°)=13-12\cos(40°)$. Thus, $c=\sqrt{13-12\cos 40°}\approx 1.951$.

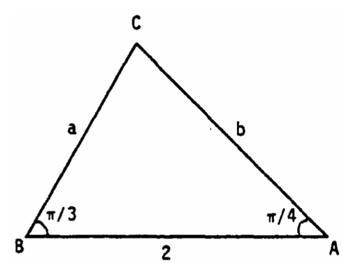
61. From the figures in the text, we see that $\sin B=\frac{h}{c}$. If C is an acute angle, then $\sin C=\frac{h}{b}$. On the other hand, if C is obtuse (as in the figure on the right in the text), then $\sin C=\sin(\pi-C)=\frac{h}{b}$. Thus, in either case, $h=b\sin C=c\sin B\Rightarrow ah=ab\sin C=ac\sin B$.

By the law of cosines, $\cos C=\frac{a^2+b^2-c^2}{2ab}$ and $\cos B=\frac{a^2+c^2-b^2}{2ac}$. Moreover, since the sum of the interior angles of triangle is π, we have $\sin A=\sin(\pi-(B+C))=\sin(B+C)=\sin B\cos C+\cos B\sin C$
$=\left(\frac{h}{c}\right)\left[\frac{a^2+b^2-c^2}{2ab}\right]+\left[\frac{a^2+c^2-b^2}{2ac}\right]\left(\frac{h}{b}\right)=\left(\frac{h}{2abc}\right)(2a^2+b^2-c^2+c^2-b^2)=\frac{ah}{bc}\Rightarrow ah=bc\sin A.$

Combining our results we have $ah=ab\sin C$, $ah=ac\sin B$, and $ah=bc\sin A$. Dividing by abc gives
$\frac{h}{bc}=\underbrace{\frac{\sin A}{a}=\frac{\sin C}{c}=\frac{\sin B}{b}}_{\text{law of sines}}.$

62. By the law of sines, $\frac{\sin A}{2}=\frac{\sin B}{3}=\frac{\sqrt{3}/2}{c}$. By Exercise 59 we know that $c=\sqrt{7}$. Thus $\sin B=\frac{3\sqrt{3}}{2\sqrt{7}}\simeq 0.982$.

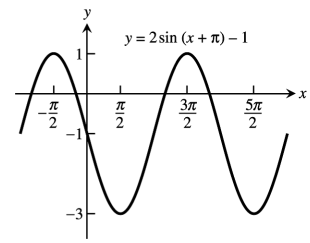
63. From the figure at the right and the law of cosines,
$b^2=a^2+2^2-2(2a)\cos B$
$=a^2+4-4a\left(\frac{1}{2}\right)=a^2-2a+4.$



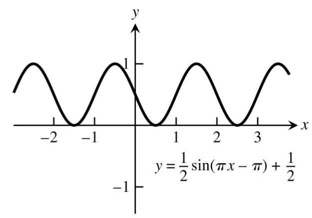
Applying the law of sines to the figure, $\frac{\sin A}{a}=\frac{\sin B}{b}$
$\Rightarrow \frac{\sqrt{2}/2}{a}=\frac{\sqrt{3}/2}{b}\Rightarrow b=\sqrt{\frac{3}{2}}\,a$. Thus, combining results,
$a^2-2a+4=b^2=\frac{3}{2}a^2\Rightarrow 0=\frac{1}{2}a^2+2a-4\Rightarrow 0=a^2+4a-8$. From the quadratic formula and the fact that $a>0$, we have $a=\frac{-4+\sqrt{4^2-4(1)(-8)}}{2}=\frac{4\sqrt{3}-4}{2}\simeq 1.464.$

64. (a) The graphs of $y=\sin x$ and $y=x$ nearly coincide when x is near the origin (when the calculator is in radians mode).
(b) In degree mode, when x is near zero degrees the sine of x is much closer to zero than x itself. The curves look like intersecting straight lines near the origin when the calculator is in degree mode.

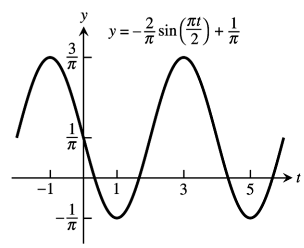
65. $A = 2, B = 2\pi, C = -\pi, D = -1$



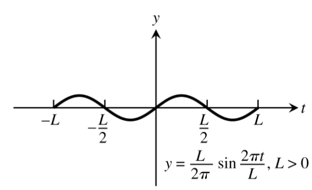
66. $A = \frac{1}{2}, B = 2, C = 1, D = \frac{1}{2}$



67. $A = -\frac{2}{\pi}, B = 4, C = 0, D = \frac{1}{\pi}$



68. $A = \frac{L}{2\pi}, B = L, C = 0, D = 0$



69–72. Example CAS commands:

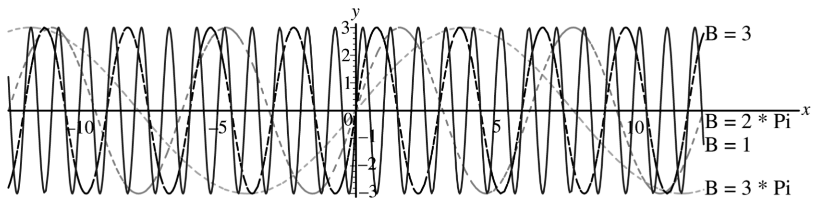
Maple:

```
f :=x -> A*sin((2*Pi/B)*(x-C))+D1;
A:=3; C:=0; D1:=0;
f_list := [seq(f(x), B=[1,3,2*Pi,5*Pi])];
plot(f_list, x=-4*Pi..4*Pi, scaling=constrained,
    color=[red,blue,green,cyan], linestyle=[1,3,4,7],
    legend=["B=1", "B=3","B=2*Pi","B=3*Pi"],
    title="#69 (Section 1.3)");
```

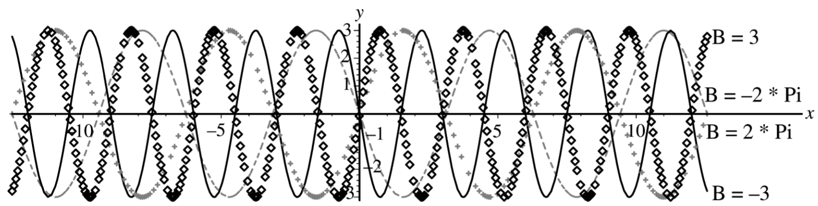
Mathematica:

```
Clear[a, b, c, d, f, x]
f[x_]:=a Sin[2π/b (x − c)] + d
Plot[f[x]/.{a → 3, b → 1, c → 0, d → 0}, {x, −4π, 4π}]
```

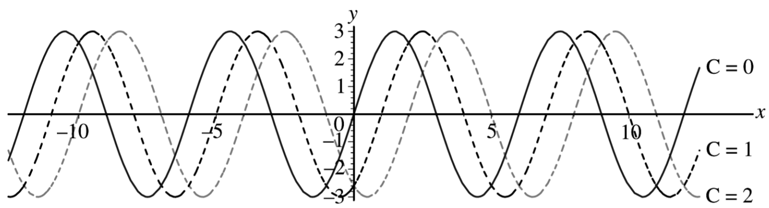
69. (a) The graph stretches horizontally.



(b) The period remains the same: period $= |B|$. The graph has a horizontal shift of $\frac{1}{2}$ period.

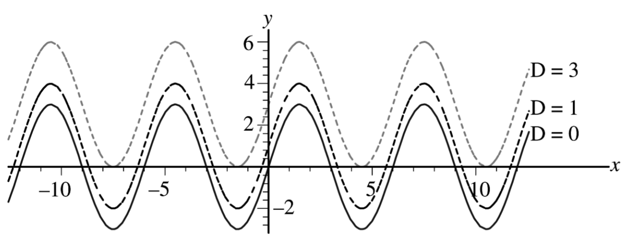


70. (a) The graph is shifted right C units.

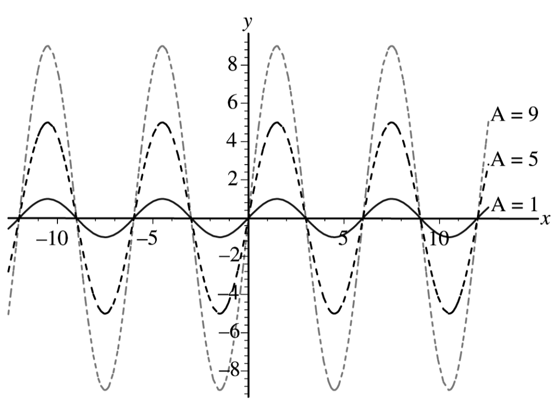


(b) The graph is shifted left C units.
(c) A shift of ± one period will produce no apparent shift. $|C| = 6$

71. (a) The graph shifts upwards $|D|$ units for $D > 0$
(b) The graph shifts down $|D|$ units for $D < 0$.



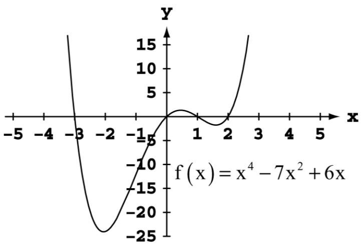
72. (a) The graph stretches $|A|$ units.
(b) For $A < 0$, the graph is inverted.



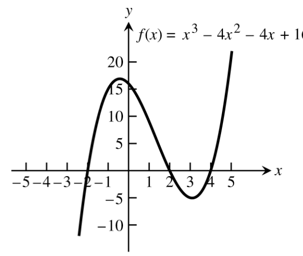
1.4 GRAPHING WITH SOFTWARE

1–4. The most appropriate viewing window displays the maxima, minima, intercepts, and end behavior of the graphs and has little unused space.

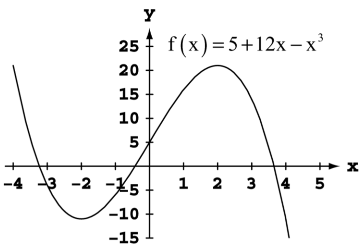
1. d.



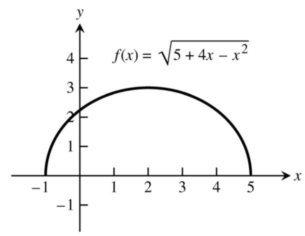
2. c.



3. d.

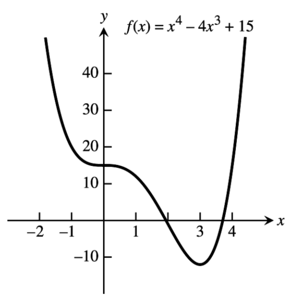


4. b.

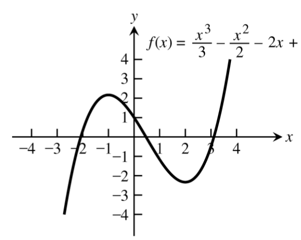


5–30. For any display there are many appropriate display widows. The graphs given as answers in Exercises 5–30 are not unique in appearance.

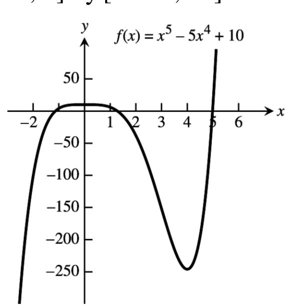
5. $[-2, 5]$ by $[-15, 40]$



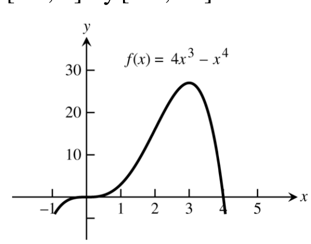
6. $[-4, 4]$ by $[-4, 4]$



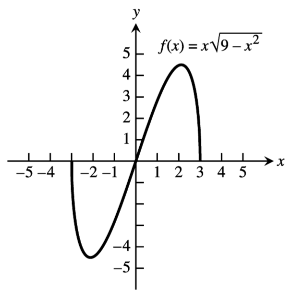
7. $[-2, 6]$ by $[-250, 50]$



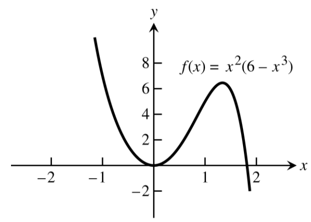
8. $[-1, 5]$ by $[-5, 30]$



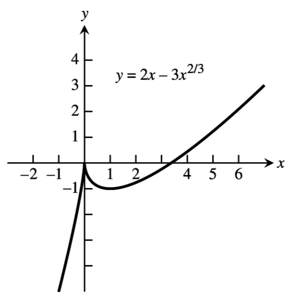
9. $[-4, 4]$ by $[-5, 5]$



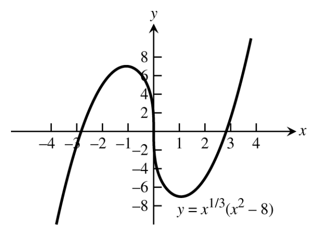
10. $[-2, 2]$ by $[-2, 8]$



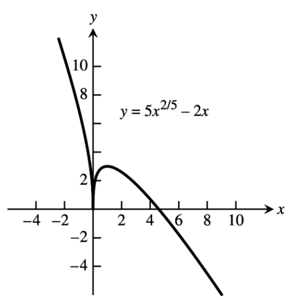
11. $[-2, 6]$ by $[-5, 4]$



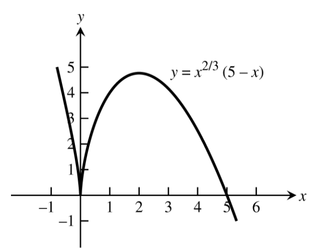
12. $[-4, 4]$ by $[-8, 8]$



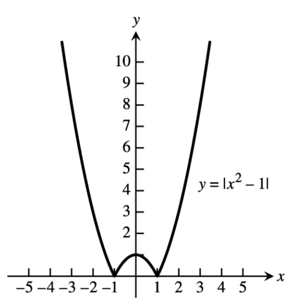
13. $[-1, 6]$ by $[-1, 4]$



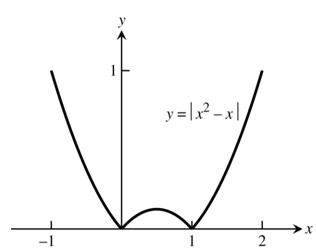
14. $[-1, 6]$ by $[-1, 5]$



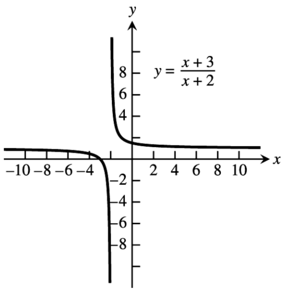
15. $[-3, 3]$ by $[0, 10]$



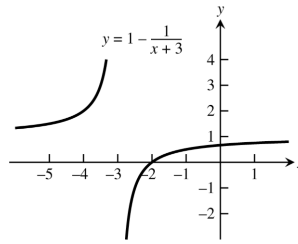
16. $[-1, 2]$ by $[0, 1]$



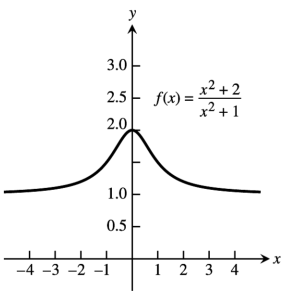
17. [−5, 1] by [−5, 5]



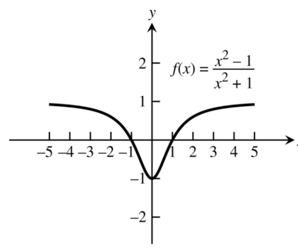
18. [−5, 1] by [−2, 4]



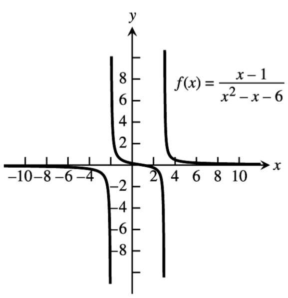
19. [−4, 4] by [0, 3]



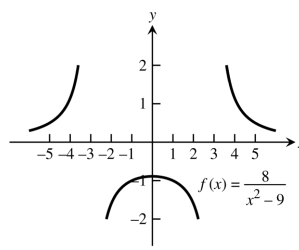
20. [−5, 5] by [−2, 2]



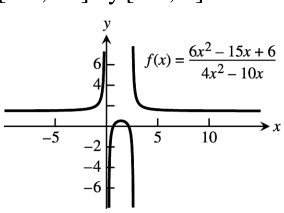
21. [−10, 10] by [−6, 6]



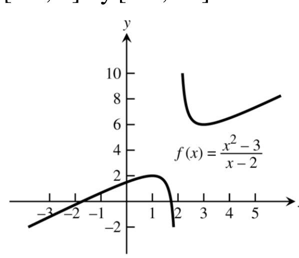
22. [−5, 5] by [−2, 2]



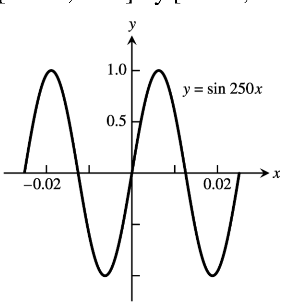
23. [−6, 10] by [−6, 6]



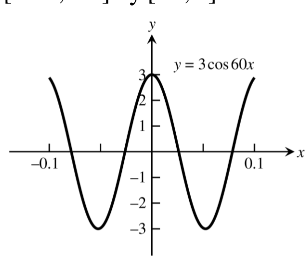
24. [−3, 5] by [−2, 10]



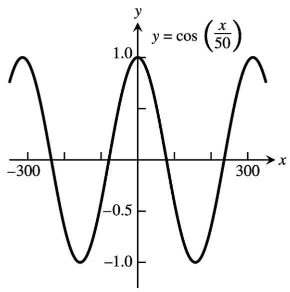
25. [−0.03, 0.03] by [−1.25, 1.25]



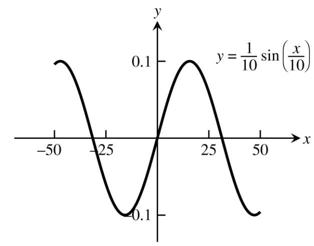
26. [−0.1, 0.1] by [−3, 3]



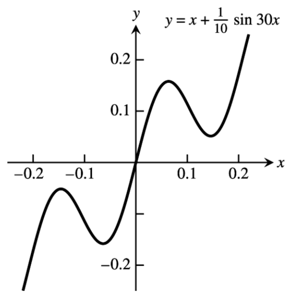
27. [–300, 300] by [–1.25, 1.25]



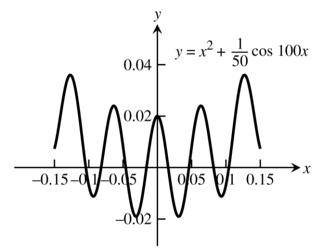
28. [–50, 50] by [–0.1, 0.1]



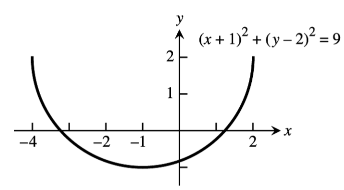
29. [–0.25, 0.25] by [–0.3, 0.3]



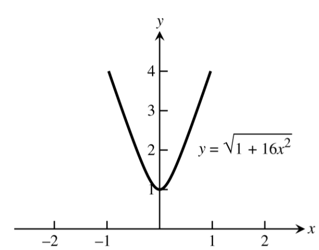
30. [–0.15, 0.15] by [–0.02, 0.05]



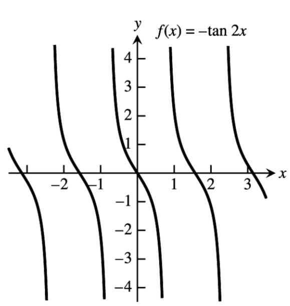
31. $x^2 + 2x = 4 + 4y - y^2 \Rightarrow y = 2 \pm \sqrt{-x^2 - 2x + 8}$.
The lower half is produced by graphing
$y = 2 - \sqrt{-x^2 - 2x + 8}$.



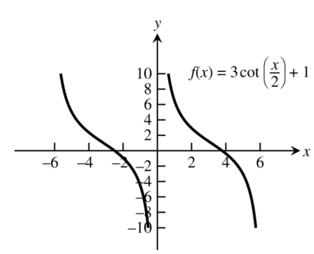
32. $y^2 - 16x^2 = 1 \Rightarrow y = \pm\sqrt{1 + 16x^2}$. The upper branch is produced by graphing $y = \sqrt{1 + 16x^2}$.



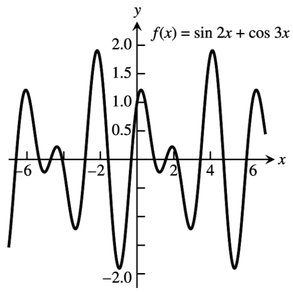
33.



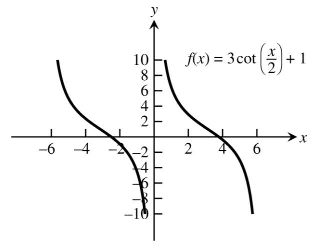
34.



35.

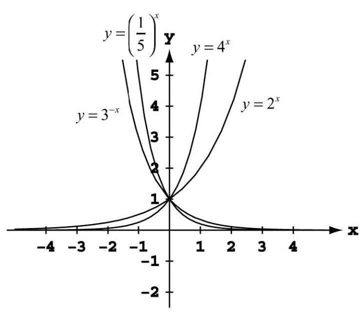


36.

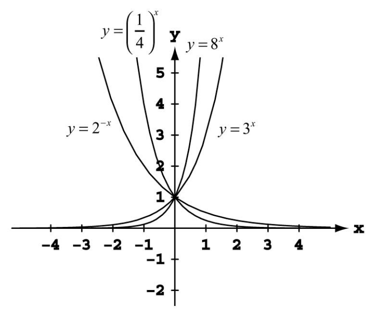


1.5 EXPONENTIAL FUNCTIONS

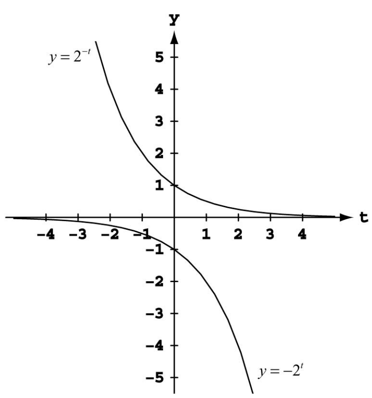
1.



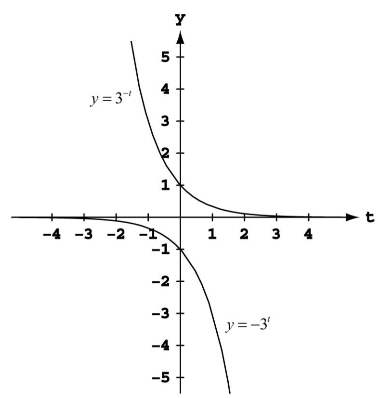
2.



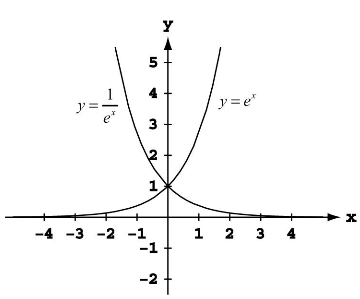
3.



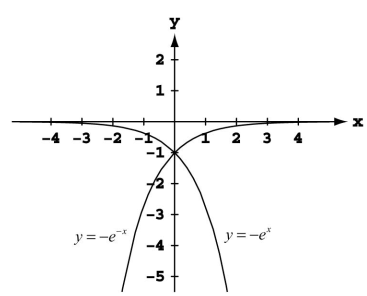
4.



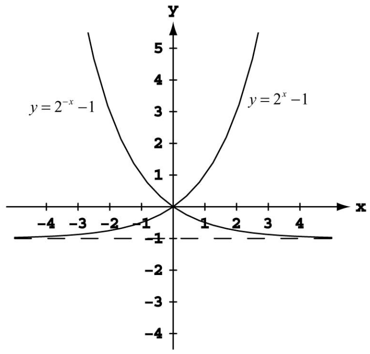
5.



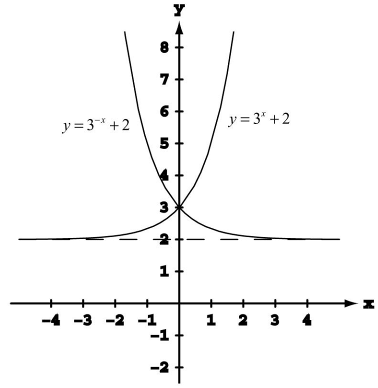
6.



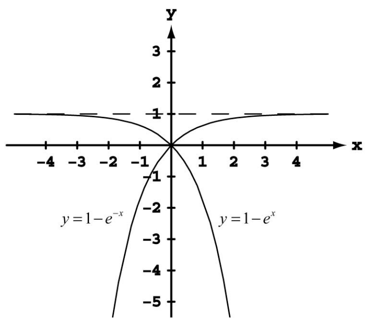
7.



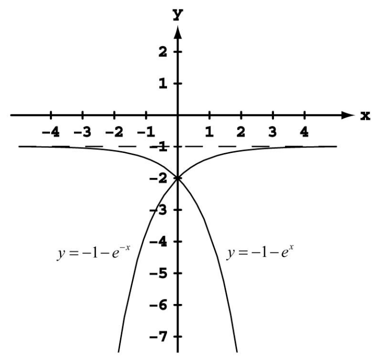
8.



9.



10.



11. $16^2 \cdot 16^{-1.75} = 16^{2+(-1.75)} = 16^{0.25} = 16^{1/4} = 2$

12. $9^{1/3} \cdot 9^{1/6} = 9^{\frac{1}{3}+\frac{1}{6}} = 9^{1/2} = 3$

13. $\frac{4^{4.2}}{4^{3.7}} = 4^{4.2-3.7} = 4^{0.5} = 4^{1/2} = 2$

14. $\frac{3^{5/3}}{3^{2/3}} = 3^{\frac{5}{3}-\frac{2}{3}} = 3^1 = 3$

15. $(25^{1/8})^4 = 25^{4/8} = 25^{1/2} = 5$

16. $\left(13^{\sqrt{2}}\right)^{\sqrt{2}/2} = 13^{2/2} = 13$

17. $2^{\sqrt{3}} \cdot 7^{\sqrt{3}} = (2 \cdot 7)^{\sqrt{3}} = 14^{\sqrt{3}}$

18. $\left(\sqrt{3}\right)^{1/2}\left(\sqrt{12}\right)^{1/2} = \left(\sqrt{3} \cdot \sqrt{12}\right)^{1/2} = \left(\sqrt{36}\right)^{1/2} = 6^{1/2}$

19. $\left(\frac{2}{\sqrt{2}}\right)^4 = \frac{2^4}{(2^{1/2})^4} = \frac{16}{2^2} = 4$

20. $\left(\frac{\sqrt{6}}{3}\right)^2 = \frac{(6^{1/2})^2}{3^2} = \frac{6}{9} = \frac{2}{3}$

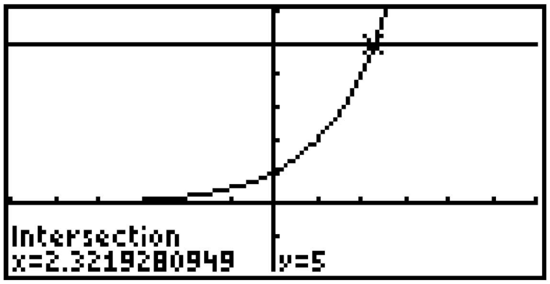
21. Domain: $(-\infty, \infty)$; y in range $\Rightarrow y = \frac{1}{2+e^x}$. As x increases, e^x becomes infinitely large and y becomes a smaller and smaller positive real number. As x decreases, e^x becomes a smaller and smaller positive real number, $y < \frac{1}{2}$, and y gets arbitrarily close to $\frac{1}{2} \Rightarrow$ Range: $\left(0, \frac{1}{2}\right)$.

22. Domain: $(-\infty, \infty)$; y in range $\Rightarrow y = \cos(e^{-t})$. Since the values of e^{-t} are $(0, \infty)$ and $-1 \le \cos x \le 1 \Rightarrow$ Range: $[-1, 1]$.

23. Domain: $(-\infty, \infty)$; y in range $\Rightarrow$ $y = \sqrt{1+3^{-t}}$. Since the values of 3^{-t} are $(0, \infty) \Rightarrow$ Range: $(1, \infty)$.

24. If $e^{2x} = 1$, then $x = 0 \Rightarrow$ Domain: $(-\infty, 0) \cup (0, \infty)$; y in range $\Rightarrow$ $y = \frac{3}{1-e^{2x}}$. If $x > 0$, then $1 < e^{2x} < \infty$ $\Rightarrow -\infty < y < 0$. If $x < 0$, then $0 < e^{2x} < 1 \Rightarrow 3 < y < \infty \Rightarrow$ Range: $(-\infty, 0) \cup (3, \infty)$.

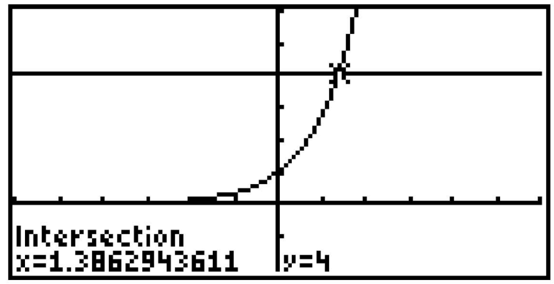
25.



[-6, 6] by [-2, 6]

$x \approx 2.3219$

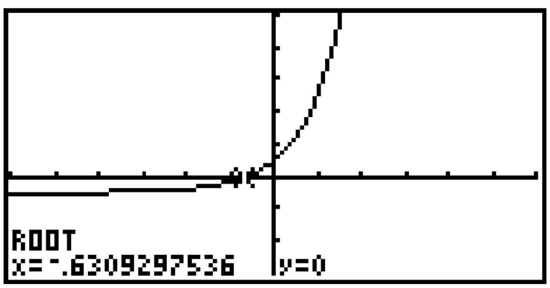
26.



[-6, 6] by [-2, 6]

$x \approx 1.3863$

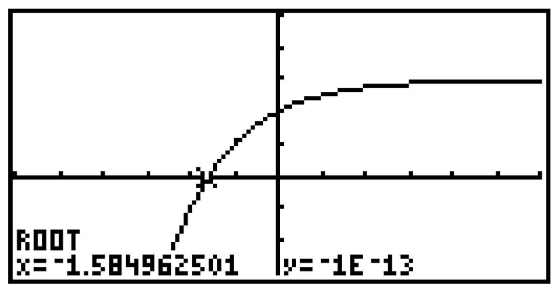
27.



[-6, 6] by [-3, 5]

$x \approx -0.6309$

28.



[-6, 6] by [-3, 5]

$x \approx -1.5850$

29. Let t be the number of years. Solving $500{,}000(1.0375)^t = 1{,}000{,}000$ graphically, we find that $t \approx 18.828$. The population will reach 1 million in about 19 years.

30. (a) The population is given by $P(t) = 6250(1.0275)^t$, where t is the number of years after 1890.
Population in 1915: $P(25) \approx 12{,}315$
Population in 1940: $P(50) \approx 24{,}265$

(b) Solving $P(t) = 50{,}000$ graphically, we find that $t \approx 76.651$. The population reached 50,000 about 77 years after 1890, in 1967.

31. (a) $A(t) = 6.6\left(\frac{1}{2}\right)^{t/14}$

(b) Solving $A(t) = 1$ graphically, we find that $t \approx 38$. There will be 1 gram remaining after about 38.1145 days.

32. Let t be the number of years. Solving $2300(1.60)^t = 4150$ graphically, we find that $t \approx 10.129$. It will take about 10.129 years. (If the interest is not credited to the account until the end of each year, it will take 11 years.)

33. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $A(1.0625)^t = 2A$, which is equivalent to $1.0625^t = 2$. Solving graphically, we find that $t \approx 11.433$. It will take about 11.433 years. (If the interest is credited at the end of each year, it will take 12 years.)

34. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $Ae^{0.0575t} = 3A$, which is equivalent to $e^{0.0575t} = 3$. Solving graphically, we find that $t \approx 19.106$. It will take about 19.106 years.

35. After t hours, the population is $P(t) = 2^{t/0.5}$, or equivalently, $P(t) = 2^{2t}$. After 24 hours, the population is $P(24) = 2^{48} \approx 2.815 \times 10^{14}$ bacteria.

36. (a) Each year, the number of cases is 100% − 20% = 80% of the previous year's number of cases. After t years, the number of cases will be $C(t) = 10{,}000(0.8)^t$. Solving $C(t) = 1000$ graphically, we find that $t \approx 10.319$. It will take 10.319 years.
    (b) Solving $C(t) = 1$ graphically, we find that $t \approx 41.275$. It will take about 41.275 years.

1.6 INVERSE FUNCTIONS AND LOGARITHMS

1. Yes one-to-one, the graph passes the horizontal line test.

2. Not one-to-one, the graph fails the horizontal line test.

3. Not one-to-one since (for example) the horizontal line $y = 2$ intersects the graph twice.

4. Not one-to-one, the graph fails the horizontal line test.

5. Yes one-to-one, the graph passes the horizontal line test.

6. Yes one-to-one, the graph passes the horizontal line test.

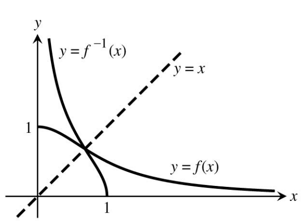
7. Not one-to-one since the horizontal line $y = 3$ intersects the graph an infinite number of times.

8. Yes one-to-one, the graph passes the horizontal line test.

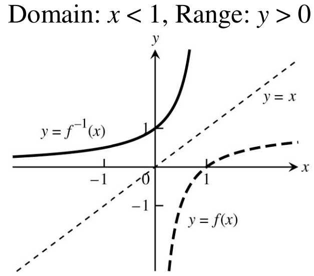
9. Yes one-to one, graph passes the horizontal line test.

10. Not one-to-one since (for example) the horizontal line $y = 1$ intersects the graph twice.

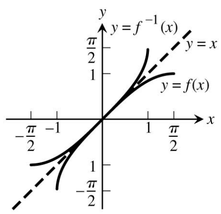
11. Domain: $0 < x \le 1$, Range: $0 \le y$



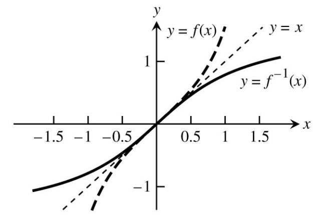
12. Domain: $x < 1$, Range: $y > 0$



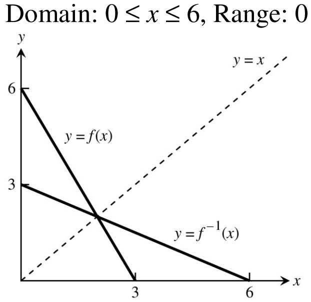
13. Domain: $-1 \le x \le 1$, Range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$



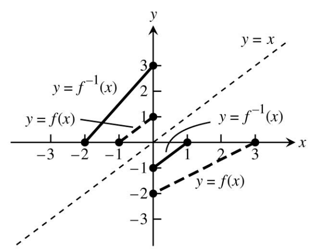
14. Domain: $-\infty < x < \infty$, Range: $-\frac{\pi}{2} < y \le \frac{\pi}{2}$



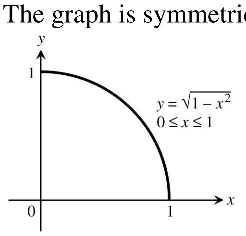
15. Domain: $0 \le x \le 6$, Range: $0 \le y \le 3$



16. Domain: $-2 \le x \le 1$, Range: $-1 \le y < 3$



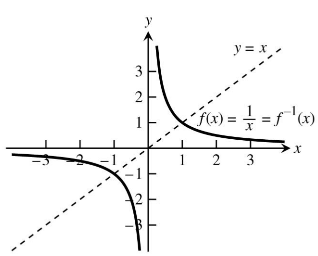
17. The graph is symmetric about $y = x$.



(b) $y = \sqrt{1-x^2} \Rightarrow y^2 = 1-x^2 \Rightarrow x^2 = 1-y^2 \Rightarrow x = \sqrt{1-y^2} \Rightarrow y = \sqrt{1-x^2} = f^{-1}(x)$

18. (a) The graph is symmetric about $y = x$.

(b) $y = \frac{1}{x} \Rightarrow x = \frac{1}{y} \Rightarrow y = \frac{1}{x} = f^{-1}(x)$



19. Step 1: $y = x^2 + 1 \Rightarrow x^2 = y - 1 \Rightarrow x = \sqrt{y-1}$

Step 2: $y = \sqrt{x-1} = f^{-1}(x)$

20. Step 1: $y = x^2 \Rightarrow x = -\sqrt{y}$, since $x \le 0$.

Step 2: $y = -\sqrt{x} = f^{-1}(x)$

21. Step 1: $y = x^3 - 1 \Rightarrow x^3 = y + 1 \Rightarrow x = (y+1)^{1/3}$

Step 2: $y = \sqrt[3]{x+1} = f^{-1}(x)$

22. Step 1: $y = x^2 - 2x + 1 \Rightarrow y = (x-1)^2 \Rightarrow \sqrt{y} = x - 1,$ since $x \ge 1 \Rightarrow x = 1 + \sqrt{y}$

Step 2: $y = 1 + \sqrt{x} = f^{-1}(x)$

23. Step 1: $y = (x+1)^2 \Rightarrow \sqrt{y} = x + 1,$ since $x \ge -1 \Rightarrow x = \sqrt{y} - 1$

Step 2: $y = \sqrt{x} - 1 = f^{-1}(x)$

24. Step 1: $y = x^{2/3} \Rightarrow x = y^{3/2}$

Step 2: $y = x^{3/2} = f^{-1}(x)$

25. Step 1: $y = x^5 \Rightarrow x = y^{1/5}$

Step 2: $y = \sqrt[5]{x} = f^{-1}(x);$

Domain and Range of f^{-1}: all reals;

$f(f^{-1}(x)) = (x^{1/5})^5 = x$ and $f^{-1}(f(x)) = (x^5)^{1/5} = x$

26. Step 1: $y = x^4 \Rightarrow x = y^{1/4}$

Step 2: $y = \sqrt[4]{x} = f^{-1}(x);$

Domain of f^{-1}: $x \ge 0$, Range of f^{-1}: $y \ge 0$;

$f(f^{-1}(x)) = (x^{1/4})^4 = x$ and $f^{-1}(f(x)) = (x^4)^{1/4} = x$

27. Step 1: $y = x^3 + 1 \Rightarrow x^3 = y - 1 \Rightarrow x = (y-1)^{1/3}$

Step 2: $y = \sqrt[3]{x-1} = f^{-1}(x);$

Domain and Range of f^{-1}: all reals;

$f(f^{-1}(x)) = ((x-1)^{1/3})^3 + 1 = (x-1) + 1 = x$ and $f^{-1}(f(x)) = ((x^3+1)-1)^{1/3} = (x^3)^{1/3} = x$

28. Step 1: $y = \frac{1}{2}x - \frac{7}{2} \Rightarrow \frac{1}{2}x = y + \frac{7}{2} \Rightarrow x = 2y + 7$

Step 2: $y = 2x + 7 = f^{-1}(x);$

Domain and Range of f^{-1}: all reals;

$f(f^{-1}(x)) = \frac{1}{2}(2x+7) - \frac{7}{2} = \left(x + \frac{7}{2}\right) - \frac{7}{2} = x$ and $f^{-1}(f(x)) = 2\left(\frac{1}{2}x - \frac{7}{2}\right) + 7 = (x-7) + 7 = x$

29. Step 1: $y = \frac{1}{x^2} \Rightarrow x^2 = \frac{1}{y} \Rightarrow x = \frac{1}{\sqrt{y}}$

Step 2: $y = \frac{1}{\sqrt{x}} = f^{-1}(x)$

Domain of f^{-1}: $x > 0$, Range of f^{-1}: $y > 0$;

$f(f^{-1}(x)) = \dfrac{1}{\left(\frac{1}{\sqrt{x}}\right)^2} = \dfrac{1}{\left(\frac{1}{x}\right)} = x$ and $f^{-1}(f(x)) = \dfrac{1}{\sqrt{\frac{1}{x^2}}} = \dfrac{1}{\left(\frac{1}{x}\right)} = x$ since $x > 0$.

30. Step 1: $y=\frac{1}{x^3}\Rightarrow x^3=\frac{1}{y}\Rightarrow x=\frac{1}{y^{1/3}}$

Step 2: $y=\frac{1}{x^{1/3}}=\sqrt[3]{\frac{1}{x}}=f^{-1}(x)$;

Domain of f^{-1}: $x\neq 0$, Range of f^{-1}: $y\neq 0$;

$f(f^{-1}(x))=\frac{1}{(x^{-1/3})^3}=\frac{1}{x^{-1}}=x$ and $f^{-1}(f(x))=\left(\frac{1}{x^3}\right)^{-1/3}=\left(\frac{1}{x}\right)^{-1}=x$

31. Step 1: $y=\frac{x+3}{x-2}\Rightarrow y(x-2)=x+3\Rightarrow xy-2y=x+3\Rightarrow xy-x=2y+3\Rightarrow x=\frac{2y+3}{y-1}$

Step 2: $y=\frac{2x+3}{x-1}=f^{-1}(x)$;

Domain of f^{-1}: $x\neq 1$, Range of f^{-1}: $y\neq 2$;

$f(f^{-1}(x))=\frac{\left(\frac{2x+3}{x-1}\right)+3}{\left(\frac{2x+3}{x-1}\right)-2}=\frac{(2x+3)+3(x-1)}{(2x+3)-2(x-1)}=\frac{5x}{5}=x$ and $f^{-1}(f(x))=\frac{2\left(\frac{x+3}{x-2}\right)+3}{\left(\frac{x+3}{x-2}\right)-1}=\frac{2(x+3)+3(x-2)}{(x+3)-(x-2)}=\frac{5x}{5}=x$

32. Step 1: $y=\frac{\sqrt{x}}{\sqrt{x}-3}\Rightarrow y\left(\sqrt{x}-3\right)=\sqrt{x}\Rightarrow y\sqrt{x}-3y=\sqrt{x}\Rightarrow y\sqrt{x}-\sqrt{x}=3y\Rightarrow x=\left(\frac{3y}{y-1}\right)^2$

Step 2: $y=\left(\frac{3x}{x-1}\right)^2=f^{-1}(x)$;

Domain of f^{-1}: $(-\infty, 0]\cup(1,\infty)$, Range of f^{-1}: $[0, 9)\cup(9,\infty)$;

$f(f^{-1}(x))=\frac{\sqrt{\left(\frac{3x}{x-1}\right)^2}}{\sqrt{\left(\frac{3x}{x-1}\right)^2}-3}$; If $x>1$ or $x\le 0\Rightarrow \frac{3x}{x-1}\ge 0\Rightarrow \frac{\sqrt{\left(\frac{3x}{x-1}\right)^2}}{\sqrt{\left(\frac{3x}{x-1}\right)^2}-3}=\frac{\frac{3x}{x-1}}{\frac{3x}{x-1}-3}=\frac{3x}{3x-3(x-1)}=\frac{3x}{3}=x$ and

$f^{-1}(f(x))=\left(\frac{3\left(\frac{\sqrt{x}}{\sqrt{x}-3}\right)}{\left(\frac{\sqrt{x}}{\sqrt{x}-3}\right)-1}\right)^2=\frac{9x}{\left(\sqrt{x}-\left(\sqrt{x}-3\right)\right)^2}=\frac{9x}{9}=x$

33. Step 1: $y=x^2-2x,\ x\le 1\Rightarrow y+1=(x-1)^2,\ x\le 1\Rightarrow -\sqrt{y+1}=x-1,\ x\le 1\Rightarrow x=1-\sqrt{y+1}$

Step 2: $y=1-\sqrt{x+1}=f^{-1}(x)$;

Domain of f^{-1}: $[-1,\infty)$, Range of f^{-1}: $(-\infty, 1]$;

$f(f^{-1}(x))=\left(1-\sqrt{x+1}\right)^2-2\left(1-\sqrt{x+1}\right)=1-2\sqrt{x+1}+x+1-2+2\sqrt{x+1}=x$ and

$f^{-1}(f(x))=1-\sqrt{(x^2-2x)+1},\ \ x\le 1=1-\sqrt{(x-1)^2},\ x\le 1=1-|x-1|=1-(1-x)=x$

34. Step 1: $y = (2x^3 + 1)^{1/5} \Rightarrow y^5 = 2x^3 + 1 \Rightarrow y^5 - 1 = 2x^3 \Rightarrow \frac{y^5-1}{2} = x^3 \Rightarrow x = \sqrt[3]{\frac{y^5-1}{2}}$

Step 2: $y = \sqrt[3]{\frac{x^5-1}{2}} = f^{-1}(x);$

Domain of f^{-1}: $(-\infty, \infty)$, Range of f^{-1}: $(-\infty, \infty)$;

$f(f^{-1}(x)) = \left(2\left(\sqrt[3]{\frac{x^5-1}{2}}\right)^3 + 1\right)^{1/5} = \left(2\left(\frac{x^5-1}{2}\right) + 1\right)^{1/5} = ((x^5 - 1) + 1)^{1/5} = (x^5)^{1/5} = x$ and

$f^{-1}(f(x)) = \sqrt[3]{\frac{\left[(2x^3+1)^{1/5}\right]^5 - 1}{2}} = \sqrt[3]{\frac{(2x^3+1)-1}{2}} = \sqrt[3]{\frac{2x^3}{2}} = x$

35. $y = \frac{x+b}{x-2} = \frac{x-2+2+b}{x-2} = 1 + \frac{2+b}{x-2} \quad x \quad \frac{2y+b}{y-1}$

$f^{-1}(x) = \frac{2x+b}{x-1}$

36. Since $x \le b$, $\quad x^2 - 2bx - y = 0 \quad x \quad \frac{2b \pm \sqrt{4b^2 + 4y}}{2} \quad b \quad \sqrt{b^2 \quad y}$

$x = b - \sqrt{b^2 + y} \quad f^{-1}(x) \quad b \quad \sqrt{b^2 \quad x}$

37. (a) $y = mx \Rightarrow x = \frac{1}{m}y \Rightarrow f^{-1}(x) = \frac{1}{m}x$

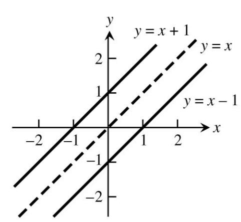
(b) The graph of $y = f^{-1}(x)$ is a line through the origin with slope $\frac{1}{m}$.

38. $y = mx + b \Rightarrow x = \frac{y}{m} - \frac{b}{m} \Rightarrow f^{-1}(x) = \frac{1}{m}x - \frac{b}{m}$; the graph of $f^{-1}(x)$ is a line with slope $\frac{1}{m}$ and y-intercept $-\frac{b}{m}$.

39. (a) $y = x + 1 \Rightarrow x = y - 1 \Rightarrow f^{-1}(x) = x - 1$

(b) $y = x + b \Rightarrow x = y - b \Rightarrow f^{-1}(x) = x - b$

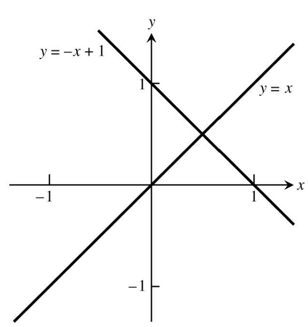
(c) Their graphs will be parallel to one another and lie on opposite sides of the line $y = x$ equidistant from that line.



40. (a) $y = -x + 1 \Rightarrow x = -y + 1 \Rightarrow f^{-1}(x) = 1 - x;$ the lines intersect at a right angle

(b) $y = -x + b \Rightarrow x = -y + b \Rightarrow f^{-1}(x) = b - x;$ the lines intersect at a right angle

(c) Such a function is its own inverse



41. (a) $\ln 0.75 = \ln \frac{3}{4} = \ln 3 - \ln 4 = \ln 3 - \ln 2^2 = \ln 3 - 2\ln 2$
(b) $\ln \frac{4}{9} = \ln 4 - \ln 9 = \ln 2^2 - \ln 3^2 = 2\ln 2 - 2\ln 3$
(c) $\ln \frac{1}{2} = \ln 1 - \ln 2 = -\ln 2$
(d) $\ln \sqrt[3]{9} = \frac{1}{3}\ln 9 = \frac{1}{3}\ln 3^2 = \frac{2}{3}\ln 3$
(e) $\ln 3\sqrt{2} = \ln 3 + \ln 2^{1/2} = \ln 3 + \frac{1}{2}\ln 2$
(f) $\ln \sqrt{13.5} = \frac{1}{2}\ln 13.5 = \frac{1}{2}\ln \frac{27}{2} = \frac{1}{2}(\ln 3^3 - \ln 2) = \frac{1}{2}(3\ln 3 - \ln 2)$

42. (a) $\ln \frac{1}{125} = \ln 1 - 3\ln 5 = -3\ln 5$
(b) $\ln 9.8 = \ln \frac{49}{5} = \ln 7^2 - \ln 5 = 2\ln 7 - \ln 5$
(c) $\ln 7\sqrt{7} = \ln 7^{3/2} = \frac{3}{2}\ln 7$
(d) $\ln 1225 = \ln 35^2 = 2\ln 35 = 2\ln 5 + 2\ln 7$
(e) $\ln 0.056 - \ln \frac{7}{125} = \ln 7 - \ln 5^3 = \ln 7 - 3\ln 5$
(f) $\frac{\ln 35 + \ln \frac{1}{7}}{\ln 25} = \frac{\ln 5 + \ln 7 - \ln 7}{2\ln 5} = \frac{1}{2}$

43. (a) $\ln \sin\theta - \ln\left(\frac{\sin\theta}{5}\right) = \ln\left(\frac{\sin\theta}{\left(\frac{\sin\theta}{5}\right)}\right) = \ln 5$
(b) $\ln(3x^2 - 9x) + \ln\left(\frac{1}{3x}\right) = \ln\left(\frac{3x^2 - 9x}{3x}\right) = \ln(x-3)$
(c) $\frac{1}{2}\ln(4t^4) - \ln 2 = \ln\sqrt{4t^4} - \ln 2 = \ln 2t^2 - \ln 2 = \ln\left(\frac{2t^2}{2}\right) = \ln(t^2)$

44. (a) $\ln \sec\theta + \ln \cos\theta = \ln[(\sec\theta)(\cos\theta)] = \ln 1 = 0$
(b) $\ln(8x+4) - \ln 2^2 = \ln(8x+4) - \ln 4 = \ln\left(\frac{8x+4}{4}\right) = \ln(2x+1)$
(c) $3\ln\sqrt[3]{t^2-1} - \ln(t+1) = 3\ln(t^2-1)^{1/3} - \ln(t+1) = 3\left(\frac{1}{3}\right)\ln(t^2-1) - \ln(t+1) = \ln\left(\frac{(t+1)(t-1)}{(t+1)}\right) = \ln(t-1)$

45. (a) $e^{\ln 7.2} = 7.2$
(b) $e^{-\ln x^2} = \frac{1}{e^{\ln x^2}} = \frac{1}{x^2}$
(c) $e^{\ln x - \ln y} = e^{\ln(x/y)} = \frac{x}{y}$

46. (a) $e^{\ln(x^2+y^2)} = x^2 + y^2$
(b) $e^{-\ln 0.3} = \frac{1}{e^{\ln 0.3}} = \frac{1}{0.3}$
(c) $e^{\ln \pi x - \ln 2} = e^{\ln(\pi x/2)} = \frac{\pi x}{2}$

47. (a) $2\ln\sqrt{e} = 2\ln e^{1/2} = (2)\left(\frac{1}{2}\right)\ln e = 1$
(b) $\ln(\ln e^e) = \ln(e\ln e) = \ln e = 1$
(c) $\ln e^{(-x^2-y^2)} = (-x^2 - y^2)\ln e = -x^2 - y^2$

48. (a) $\ln\left(e^{\sec\theta}\right) = (\sec\theta)(\ln e) = \sec\theta$
(b) $\ln e^{(e^x)} = (e^x)(\ln e) = e^x$
(c) $\ln(e^{2\ln x}) = \ln\left(e^{\ln x^2}\right) = \ln x^2 = 2\ln x$

49. $\ln y = 2t + 4 \Rightarrow e^{\ln y} = e^{2t+4} \Rightarrow y = e^{2t+4}$

50. $\ln y = -t + 5 \Rightarrow e^{\ln y} = e^{-t+5} \Rightarrow y = e^{-t+5}$

51. $\ln(y - 40) = 5t \Rightarrow e^{\ln(y-40)} = e^{5t} \Rightarrow y - 40 = e^{5t} \Rightarrow y = e^{5t} + 40$

52. $\ln(1 - 2y) = t \Rightarrow e^{\ln(1-2y)} = e^t \Rightarrow 1 - 2y = e^t \Rightarrow -2y = e^t - 1 \Rightarrow y = -\left(\frac{e^t - 1}{2}\right)$

53. $\ln(y-1) - \ln 2 = x + \ln x \Rightarrow \ln(y-1) - \ln 2 - \ln x = x \Rightarrow \ln\left(\frac{y-1}{2x}\right) = x \Rightarrow e^{\ln\left(\frac{y-1}{2x}\right)} = e^x \Rightarrow \frac{y-1}{2x} = e^x$

$\Rightarrow y-1 = 2xe^x \Rightarrow y = 2xe^x + 1$

54. $\ln(y^2-1) - \ln(y+1) = \ln(\sin x) \Rightarrow \ln\left(\frac{y^2-1}{y+1}\right) = \ln(\sin x) \Rightarrow \ln(y-1) = \ln(\sin x) \Rightarrow e^{\ln(y-1)} = e^{\ln(\sin x)}$

$\Rightarrow y - 1 = \sin x \Rightarrow y = \sin x + 1$

55. (a) $e^{2k} = 4 \Rightarrow \ln e^{2k} = \ln 4 \Rightarrow 2k \ln e = \ln 2^2 \Rightarrow 2k = 2 \ln 2 \Rightarrow k = \ln 2$
(b) $100e^{10k} = 200 \Rightarrow e^{10k} = 2 \Rightarrow \ln e^{10k} = \ln 2 \Rightarrow 10k \ln e = \ln 2 \Rightarrow 10k = \ln 2 \Rightarrow k = \frac{\ln 2}{10}$
(c) $e^{k/1000} = a \Rightarrow \ln e^{k/1000} = \ln a \Rightarrow \frac{k}{1000} \ln e = \ln a \Rightarrow \frac{k}{1000} = \ln a \Rightarrow k = 1000 \ln a$

56. (a) $e^{5k} = \frac{1}{4} \Rightarrow \ln e^{5k} = \ln 4^{-1} \Rightarrow 5k \ln e = -\ln 4 \Rightarrow 5k = -\ln 4 \Rightarrow k = -\frac{\ln 4}{5}$
(b) $80e^k = 1 \Rightarrow e^k = 80^{-1} \Rightarrow \ln e^k = \ln 80^{-1} \Rightarrow k \ln e = -\ln 80 \Rightarrow k = -\ln 80$
(c) $e^{(\ln 0.8)k} = 0.8 \Rightarrow (e^{\ln 0.8})^k = 0.8 \Rightarrow (0.8)^k = 0.8 \Rightarrow k = 1$

57. (a) $e^{-0.3t} = 27 \Rightarrow \ln e^{-0.3t} = \ln 3^3 \Rightarrow (-0.3t)\ln e = 3 \ln 3 \Rightarrow -0.3t = 3 \ln 3 \Rightarrow t = -10 \ln 3$
(b) $e^{kt} = \frac{1}{2} \Rightarrow \ln e^{kt} = \ln 2^{-1} = kt \ln e = -\ln 2 \Rightarrow t = -\frac{\ln 2}{k}$
(c) $e^{(\ln 0.2)t} = 0.4 \Rightarrow (e^{\ln 0.2})^t = 0.4 \Rightarrow 0.2^t = 0.4 \Rightarrow \ln 0.2^t = \ln 0.4 \Rightarrow t \ln 0.2 = \ln 0.4 \Rightarrow t = \frac{\ln 0.4}{\ln 0.2}$

58. (a) $e^{-0.01t} = 1000 \Rightarrow \ln e^{-0.01t} = \ln 1000 \Rightarrow (-0.01\text{t})\ln e = \ln 1000 \Rightarrow -0.01t = \ln 1000 \Rightarrow t = -100 \ln 1000$
(b) $e^{kt} = \frac{1}{10} \Rightarrow \ln e^{kt} = \ln 10^{-1} = kt \ln e = -\ln 10 \Rightarrow kt = -\ln 10 \Rightarrow t = -\frac{\ln 10}{k}$
(c) $e^{(\ln 2)t} = \frac{1}{2} \Rightarrow (e^{\ln 2})^t = 2^{-1} \Rightarrow 2^t = 2^{-1} \Rightarrow t = -1$

59. $e^{\sqrt{t}} = x^2 \Rightarrow \ln e^{\sqrt{t}} = \ln x^2 \Rightarrow \sqrt{t} = 2 \ln x \Rightarrow t = 4(\ln x)^2$

60. $e^{x^2} e^{2x+1} = e^t \Rightarrow e^{x^2+2x+1} = e^t \Rightarrow \ln e^{x^2+2x+1} = \ln e^t \Rightarrow t = x^2 + 2x + 1$

61. (a) $5^{\log_5 7} = 7$ (b) $8^{\log_8 \sqrt{2}} = \sqrt{2}$ (c) $1.3^{\log_3 75} = 75$
(d) $\log_4 16 = \log_4 4^2 = 2 \log_4 4 = 2 \cdot 1 = 2$
(e) $\log_3 \sqrt{3} = \log_3 3^{1/2} = \frac{1}{2} \log_3 3 = \frac{1}{2} \cdot 1 = \frac{1}{2} = 0.5$
(f) $\log_4\left(\frac{1}{4}\right) = \log_4 4^{-1} = -1 \log_4 4 = -1 \cdot 1 = -1$

62. (a) $2^{\log_2 3} = 3$ (b) $10^{\log_{10}(1/2)} = \frac{1}{2}$ (c) $\pi^{\log_\pi 7} = 7$

(d) $\log_{11} 121 = \log_{11} 11^2 = 2\log_{11} 11 = 2 \cdot 1 = 2$

(e) $\log_{121} 11 = \log_{121} 121^{1/2} = \left(\frac{1}{2}\right)\log_{121} 121 = \left(\frac{1}{2}\right) \cdot 1 = \frac{1}{2}$

(f) $\log_3\left(\frac{1}{9}\right) = \log_3 3^{-2} = -2\log_3 3 = -2 \cdot 1 = -2$

63. (a) Let $z = \log_4 x \Rightarrow 4^z = x \Rightarrow 2^{2z} = x \Rightarrow (2^z)^2 = x \Rightarrow 2^z = \sqrt{x}$

(b) Let $z = \log_3 x \Rightarrow 3^z = x \Rightarrow (3^z)^2 = x^2 \Rightarrow 3^{2z} = x^2 \Rightarrow 9^z = x^2$

(c) $\log_2(e^{(\ln 2)\sin x}) = \log_2 2^{\sin x} = \sin x$

64. (a) Let $z = \log_5(3x^2) \Rightarrow 5^z = 3x^2 \Rightarrow 25^z = 9x^4$

(b) $\log_e(e^x) = x$

(c) $\log_4(2^{e^x \sin x}) = \log_4 4^{(e^x \sin x)/2} = \frac{e^x \sin x}{2}$

65. (a) $\frac{\log_2 x}{\log_3 x} = \frac{\ln x}{\ln 2} \div \frac{\ln x}{\ln 3} = \frac{\ln x}{\ln 2} \cdot \frac{\ln 3}{\ln x} = \frac{\ln 3}{\ln 2}$ (b) $\frac{\log_2 x}{\log_8 x} = \frac{\ln x}{\ln 2} \div \frac{\ln x}{\ln 8} = \frac{\ln x}{\ln 2} \cdot \frac{\ln 8}{\ln x} = \frac{3\ln 2}{\ln 2} = 3$

(c) $\frac{\ln_x a}{\ln_{x^2} a} = \frac{\ln a}{\ln x} \div \frac{\ln a}{\ln x^2} = \frac{\ln a}{\ln x} \cdot \frac{\ln x^2}{\ln a} = \frac{2\ln x}{\ln x} = 2$

66. (a) $\frac{\log_9 x}{\log_3 x} = \frac{\ln x}{\ln 9} \div \frac{\ln x}{\ln 3} = \frac{\ln x}{2\ln 3} \cdot \frac{\ln 3}{\ln x} = \frac{1}{2}$

(b) $\frac{\log_{\sqrt{10}} x}{\log_{\sqrt{2}} x} = \frac{\ln x}{\ln\sqrt{10}} \div \frac{\ln x}{\ln\sqrt{2}} = \frac{\ln x}{\left(\frac{1}{2}\right)\ln 10} \cdot \frac{\left(\frac{1}{2}\right)\ln 2}{\ln x} = \frac{\ln 2}{\ln 10}$

(c) $\frac{\log_a b}{\log_b a} = \frac{\ln b}{\ln a} \div \frac{\ln a}{\ln b} = \frac{\ln b}{\ln a} \cdot \frac{\ln b}{\ln a} = \left(\frac{\ln b}{\ln a}\right)^2$

67. (a) $-\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $-\frac{\pi}{3}$

68. (a) $\frac{\pi}{3}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{6}$

69. (a) $\arccos(-1) = \pi$ since $\cos(\pi) = -1$ and $0 \le \pi \le \pi$.

(b) $\arccos(0) = \frac{\pi}{2}$ since $\cos\left(\frac{\pi}{2}\right) = 0$ and $0 \le \frac{\pi}{2} \le \pi$.

70. (a) $\arcsin(-1) = -\frac{\pi}{2}$ since $\sin\left(-\frac{\pi}{2}\right) = -1$ and $-\frac{\pi}{2} \le -\frac{\pi}{2} \le \frac{\pi}{2}$.

(b) $\arcsin\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$ since $\sin\left(-\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$ and $-\frac{\pi}{2} \le -\frac{\pi}{4} \le \frac{\pi}{2}$.

71. The function $g(x)$ is also one-to-one. The reasoning: $f(x)$ is one-to-one means that if $x_1 \ne x_2$ then $f(x_1) \ne f(x_2)$, so $-f(x_1) \ne -f(x_2)$ and therefore $g(x_1) \ne g(x_2)$. Therefore $g(x)$ is one-to-one as well.

72. The function $h(x)$ is also one-to-one. The reasoning: $f(x)$ is one-to-one means that if $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$, so $\frac{1}{f(x_1)} \neq \frac{1}{f(x_2)}$, and therefore $h(x_1) \neq h(x_2)$.

73. The composite is one-to-one also. The reasoning: If $x_1 \neq x_2$ then $g(x_1) \neq g(x_2)$ because g is one-to-one. Since $g(x_1) \neq g(x_2)$, we also have $f(g(x_1)) \neq f(g(x_2))$ because f is one-to-one; thus, $f \circ g$ is one-to-one because $x_1 \neq x_2 \Rightarrow f(g(x_1)) \neq f(g(x_2))$.

74. Yes, g must be one-to-one. If g were not one-to-one, there would exist numbers $x_1 \neq x_2$ in the domain of g with $g(x_1) = g(x_2)$. For these numbers we would also have $f(g(x_1)) = f(g(x_2))$, contradicting the assumption that $f \circ g$ is one-to-one.

75. (a) $y = \frac{100}{1+2^{-x}} \to 1+2^{-x} = \frac{100}{y} \to 2^{-x} = \frac{100}{y} - 1 \to \log_2(2^{-x}) = \log_2\left(\frac{100}{y}-1\right) \to -x = \log_2\left(\frac{100}{y}-1\right)$

$x = -\log_2\left(\frac{100}{y}-1\right) = -\log_2\left(\frac{100-y}{y}\right) = \log_2\left(\frac{y}{100-y}\right)$.

Interchange x and y: $y = \log_2\left(\frac{x}{100-x}\right) \to f^{-1}(x) = \log_2\left(\frac{x}{100-x}\right)$

Verify.

$(f \circ f^{-1})(x) = f\left(\log_2\left(\frac{x}{100-x}\right)\right) = \frac{100}{1+2^{\log_2\left(\frac{x}{100-x}\right)}} = \frac{100}{1+2^{\log_2\left(\frac{100-x}{x}\right)}} = \frac{100}{1+\frac{100-x}{x}} = \frac{100}{x+100-x} = \frac{100x}{100} = x$

$(f^{-1} \circ f)(x) = f^{-1}\left(\frac{100}{1+2^{-x}}\right) = \log_2\left(\frac{\frac{100}{1+2^{-x}}}{100-\frac{100}{1+2^{-x}}}\right) = \log_2\left(\frac{100}{100(1+2^{-x})-100}\right) = \log_2\left(\frac{1}{2^{-x}}\right) = \log_2(2^x) = x$

(b) $y = \frac{50}{1+1.1^{-x}} \to 1+1.1^{-x} = \frac{50}{y} \to 1.1^{-x} = \frac{50}{y} - 1 \to \log_{1.1}(1.1^{-x}) = \log_{1.1}\left(\frac{50}{y}-1\right) \to -x = \log_{1.1}\left(\frac{50}{y}-1\right)$

$x = -\log_{1.1}\left(\frac{50}{y}-1\right) = -\log_{1.1}\left(\frac{50-y}{y}\right) = \log_{1.1}\left(\frac{y}{50-y}\right)$.

Interchange x and y: $y = \log_{1.1}\left(\frac{x}{50-x}\right) \to f^{-1}(x) = \log_{1.1}\left(\frac{x}{50-x}\right)$

Verify.

$(f \circ f^{-1})(x) = f\left(\log_{1.1}\left(\frac{x}{50-x}\right)\right) = \frac{50}{1+1.1^{\log_{1.1}\left(\frac{x}{50-x}\right)}} = \frac{50}{1+1.1^{\log_{1.1}\left(\frac{50-x}{x}\right)}} = \frac{50}{1+\frac{50-x}{x}} = \frac{50x}{x+50-x} = \frac{50x}{50} = x$

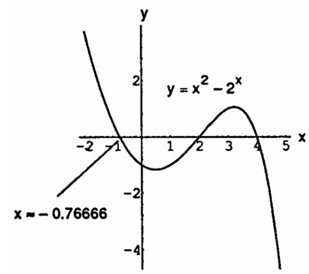
$(f^{-1} \circ f)(x) = f^{-1}\left(\frac{50}{1+1.1^{-x}}\right) = \log_{1.1}\left(\frac{\frac{50}{1+1.1^{-x}}}{50-\frac{50}{1+1.1^{-x}}}\right) = \log_{1.1}\left(\frac{50}{50(1+1.1^{-x})-50}\right) = \log_{1.1}\left(\frac{1}{1.1^{-x}}\right) = \log_{1.1}(1.1^x) = x$

76. $\sin^{-1}(1) + \cos^{-1}(1) = \frac{\pi}{2} + 0 = \frac{\pi}{2}$; $\sin^{-1}(0) + \cos^{-1}(0) = 0 + \frac{\pi}{2} = \frac{\pi}{2}$; and $\sin^{-1}(-1) + \cos^{-1}(-1) = -\frac{\pi}{2} + \pi = \frac{\pi}{2}$.
If $x \in (-1, 0)$ and $x = -a$, then $\sin^{-1}(x) + \cos^{-1}(x) = \sin^{-1}(-a) + \cos^{-1}(-a) = -\sin^{-1} a + (\pi - \cos^{-1} a)$ $= \pi - (\sin^{-1} a + \cos^{-1} a) = \pi - \frac{\pi}{2} = \frac{\pi}{2}$ from Equations (3) and (4) in the text.

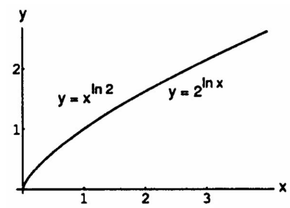
77. (a) Begin with $y = \ln x$ and reduce the y-value by $3 \Rightarrow y = \ln x - 3$.
(b) Begin with $y = \ln x$ and replace x with $x - 1 \Rightarrow y = \ln(x - 1)$.
(c) Begin with $y = \ln x$, replace x with $x + 1$, and increase the y-value by $3 \Rightarrow y = \ln(x + 1) + 3$.
(d) Begin with $y = \ln x$, reduce the y-value by 4, and replace x with $x - 2 \Rightarrow y = \ln(x - 2) - 4$.
(e) Begin with $y = \ln x$ and replace x with $-x \Rightarrow y = \ln(-x)$.
(f) Begin with $y = \ln x$ and switch x and $y \Rightarrow x = \ln y$ or $y = e^x$.

78. (a) Begin with $y = \ln x$ and multiply the y-value by $2 \Rightarrow y = 2\ln x$.
(b) Begin with $y = \ln x$ and replace x with $\frac{x}{3} \Rightarrow y = \ln\left(\frac{x}{3}\right)$.
(c) Begin with $y = \ln x$ and multiply the y-value by $\frac{1}{4} \Rightarrow y = \frac{1}{4}\ln x$.
(d) Begin with $y = \ln x$ and replace x with $2x \Rightarrow y = \ln 2x$.

79. From zooming in on the graph at the right, we estimate the third root to be $x \approx -0.76666$.



80. The functions $f(x) = x^{\ln 2}$ and $g(x) = 2^{\ln x}$ appear to have identical graphs for $x > 0$. This is no accident, because $x^{\ln 2} = e^{\ln 2 \cdot \ln x} = (e^{\ln 2})^{\ln x} = 2^{\ln x}$.



81. (a) Amount $= 8\left(\frac{1}{2}\right)^{t/12}$
(b) $8\left(\frac{1}{2}\right)^{t/12} = 1 \to \left(\frac{1}{2}\right)^{t/12} = \frac{1}{8} \to \left(\frac{1}{2}\right)^{t/12} = \left(\frac{1}{2}\right)^3 \to \frac{t}{12} = 3 \to t = 36$
There will be 1 gram remaining after 36 hours.

82. $500(1.0475)^t = 1000 \to 1.0475^t = 2 \to \ln(1.0475^t) = \ln(2) \to t\ln(1.0475) = \ln(2) \to t = \frac{\ln(2)}{\ln(1.0475)} \approx 14.936$
It will take about 14.936 years. (If the interest is paid at the end of each year, it will take 15 years.)

83. $375{,}000(1.0225)^t = 1{,}000{,}000 \to 1.0225^t = \frac{8}{3} \to \ln(1.0225^t) = \ln\left(\frac{8}{3}\right) \to t\ln(1.0225) = \ln\left(\frac{8}{3}\right)$
$\to t = \frac{\ln\left(\frac{8}{3}\right)}{\ln(1.0225)} \approx 44.081$
It will take about 44.081 years.

84. $y = y_0 e^{-0.18t}$ represents the decay equation; solving $(0.9)y_0 = y_0 e^{-0.18t} \Rightarrow t = \frac{\ln(0.9)}{-0.18} \approx 0.585$ days